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DECEMBER 1964
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Technical Memorandum

EQUILIBRIUM, ADIABATIC WALL, AND STAGNATION TEMPERATURES AT ALTITUDES UP TO 100,000 FEET AND MACH NUMBERS UP TO 20

by R. P. SUESS and L. B. WECKESSER

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THE JOHNS HOPKINS UNIVERSITY • APPLIED PHYSICS LABORATORY

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8621 Georgia Avenue, Silver Spring, Maryland 20910
Operating under Contract NOw 62-0604-c, Bureau of Naval Weapons, Department of the Navy

ABSTRACT

In the preliminary design phase of high velocity vehicles, quick, conservative estimates of surface temperatures are always desirable. To fill this need, the equilibrium, adiabatic wall, and stagnation temperatures have been determined for altitudes from sea level to 100,000 feet and for Mach numbers from 2 to 20. Data for both laminar and turbulent flow are presented. For easy reading, the results are presented as temperature versus Mach number plots at altitude intervals of 10,000 feet. The only information necessary to obtain equilibrium temperatures from this report is: distance from the leading edge to the point in question, emissivity of the surface, Mach number, and altitude.

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EQUILIBRIUM, ADIABATIC WALL, AND STAGNATION TEMPERATURES AT ALTITUDES UP TO 100,000 FEET AND MACH NUMBERS UP TO 20

INTRODUCTION

During preliminary design studies of high velocity vehicles, the need often arises for estimates of maximum temperatures to be expected at various locations on the vehicles. These estimates must be made without the knowledge of such influencing factors as materials, thicknesses, and flight trajectory all of which undoubtedly remain to be chosen. Computer transient temperature calculations are impractical because of the lack of knowledge of such factors and because those interested in the temperature estimates often do not have the time needed to perform the transient calculations.

The work presented in this report is intended to provide an easily accessible capability to predict maximum temperatures at various locations on vehicles flying over a wide range of Mach numbers and altitudes. The results present flat plate equilibrium, stagnation, and flat plate adiabatic wall temperatures as functions of Mach number and altitude in convenient graphical form.

Two sets of curves are presented; one for a laminar boundary layer and the other for a turbulent boundary layer. Mach numbers up to 20 are considered for each 10,000 foot altitude up to 100,000 feet. On each graph, several flat plate equilibrium temperatures representing various combinations of distance from the beginning of boundary layer build-up and surface emissivity are given in addition to the

stagnation and adiabatic wall temperatures. A discussion of transition Reynolds number is also included to serve as a rough guide in choosing the appropriate set of graphs to use for a given condition. The following discussion will describe the use of the graphs and briefly discuss the theories upon which they are based.

DEFINITION OF EQUILIBRIUM, ADIABATIC WALL, AND STAGNATION TEMPERATURES

Equilibrium temperature may be defined, in general, as that temperature attained by an element in a constant environment when the heat being transferred to the element is equal to that being lost by the element. In estimating the temperatures of aerodynamic surfaces, however, the general rule is to assume a perfectly insulated surface such that no conduction heat transfer need be considered. Thus, for aerodynamic surfaces, the equilibrium temperature is defined to be the temperature attained by a perfectly insulated surface when the convective heat transfer to the surface is exactly balanced by the radiation heat transfer away from the surface. Obviously then, the equilibrium temperatures represent maximum temperatures since all of the energy received at the surface must be lost from the surface by radiation.

Since thermal capacitance of the surface material and heat conduction from the surface to the material beneath is not involved in the computation of the equilibrium temperatures, under certain conditions the equilibrium temperatures are quite conservative estimates of the maximum surface temperature. This would be particularly true for a massive article subjected to a given flight environment or for a lightweight structure heated during a relatively short flight time. For example, the equilibrium temperature of a point one foot from the leading edge having a surface emissivity of 0.8 and flying at Mach 7 and 90,000 feet is 1025°F (assuming laminar flow). This same point, if assumed to be

constructed of 0.05 inch steel, would have a surface temperature of only 520°F after a 50 second flight. However, after a 100 second flight the surface temperature would reach 820°F. The purpose of citing this example is not to minimize the value of the equilibrium temperatures but to help illustrate their meaning, usefulness, and limitations.

The adiabatic wall temperature, sometimes referred to as the recovery temperature, is the temperature a surface, perfectly insulated on its back side, will reach due to convective heating in the absence of radiation relief. This temperature may be explained in the following way. At every point on a body other than the stagnation point, the air is brought to rest by viscous effects in the boundary layer. This results in a velocity gradient across the boundary layer. The temperature of the air near the wall is increased by stagnation and by a transfer of momentum toward the wall resulting from the velocity gradient. In an adiabatic system no heat is transferred through the body itself; however, the rise in temperature of the wall above the moving-stream temperature causes conduction of heat back through the gas layers near the wall into the bulk stream. Consequently, the wall assumes a temperature below the stagnation value which is referred to as the adiabatic wall temperature.

By definition, the free stream total temperature is that temperature attained by a gas when it is brought to rest isentropically. In the case of supersonic flow, however, the air must pass through a normal shock in the area of a stagnation point, thus it is not brought to rest isentropically. Since the primary interest is in temperatures applicable to flight vehicles, the stagnation values reported herein are total temperatures of the air behind a normal

shock. In this report these total temperatures are called stagnation temperatures to distinguish them from free stream total temperatures. As might be expected, at high velocities the stagnation temperatures are appreciably below the free stream total temperature due to the dissociation and ionization taking place behind the normal shock. Stagnation temperatures may be used to estimate the stagnation point wall temperature, although they will obviously yield conservative answers. Stagnation point equilibrium temperatures would be better estimates of the wall temperature but are beyond the scope of this report.

DISCUSSION OF THEORY

Due to the presence of real gas effects, it was necessary to compute the stagnation and adiabatic wall temperatures on an enthalpy basis. The stagnation temperatures were computed using the following procedure. The stagnation enthalpy was computed by the equation.

$$H_S = H_L + \frac{V_L^2}{2gJ}$$

(See Appendix A for a definition of terminology.)

By definition:

$$V_L = M_L a_L$$

Therefore:

$$H_S = H_L + \frac{M_L^2 a_L^2}{2gJ} \tag{1}$$

The procedure used to obtain the stagnation temperatures was the following:

1. For a given Mach number and altitude the total pressure behind a normal shock for a real gas was determined using Ref. 1.
2. Using Eq. (1) the stagnation enthalpy was computed.
3. Entering the Mollier Chart for equilibrium air (Ref. 2) with the pressure and enthalpy found in

steps 1 and 2, the stagnation temperature was obtained.

Since the adiabatic wall temperatures are needed to compute the equilibrium temperatures, they were both computed on the IBM 7094 computer. The adiabatic wall temperatures were computed by first obtaining the adiabatic wall enthalpies using the following equation:

$$H_{aw} = H_L + r \frac{M_L^2 a_L^2}{2gJ} \qquad (2)$$

where r = recovery factor
= 0.85 for laminar flow (Ref. 3)
= 0.90 for turbulent flow (Ref. 3).

The conversion of the enthalpies to temperatures was performed at one atmosphere of pressure, only, due to the limitation inherent in the computer program making the calculations. The influence of pressure on the enthalpy-temperature relation may be appreciable, however, for the range of Mach numbers and altitudes considered here the maximum variation from true temperature was less than 11 per cent. Because the pressures are less than atmospheric, the values presented are conservative; i.e., the temperatures presented herein are higher than those obtained using the actual pressure corresponding to each altitude.

Since the term "equilibrium temperature" is defined as the temperature reached by a perfectly insulated surface when the convective heat input to the surface is exactly balanced by the radiation relief from the surface, the equation for computing equilibrium temperatures is derived in the following manner. The convective heat transfer is defined as:

$$q_c = hA(T_{aw} - T_w) \qquad (3)$$

A term, h', may now be introduced which is independent of the reference length.

$$h' = hx^{\alpha}$$

Therefore:

$$q_c = \frac{h'}{x^{\alpha}} A(T_{aw} - T_w)$$

The radiation heat transfer is:

$$q_r = \sigma\epsilon A(T_w^4 - T_s^4)$$

By equating q_r and q_c, the wall temperature T_w becomes the equilibrium temperature T_e. Hence:

$$h'(T_{aw} - T_e) = \sigma\epsilon x^{\alpha}(T_e^4 - T_s^4)$$

If ϵx^{α} is defined by a new term, η, then:

$$h'(T_{aw} - T_e) = \sigma\eta(T_e^4 - T_s^4) \qquad (4)$$

An IBM 7094 computer program was written to solve Eq. (4) for T_e. The technique used for the solution was to guess an initial value for T_e (arbitrarily 300°R in every case) and iterate the equation until two successive identical values of T_e were computed. The computation of h', which itself is a function of T_e, was based on a reference enthalpy scheme and is described in Appendix B. The value of T_{aw} was supplied by Eq. (2) for either a laminar or turbulent boundary

layer, while the value of the space temperature, T_s, was chosen to be 0°R. Although the choice of 0°R is somewhat arbitrary, Eq. (4) is relatively insensitive to the space temperature. The final item needed by Eq. (4) to compute T_e was η. For this purpose seven different values of the parameter η were chosen and supplied to the program as inputs.

DISCUSSION OF TRANSITION REYNOLDS NUMBER

Two sets of curves are presented in this report; one for a laminar boundary layer and the other for a turbulent boundary layer. Consequently, the user of this report is confronted with a choice of which set to use for a particular set of conditions. To aid in this choice, the concept of transition Reynolds number is now introduced.

In heat transfer computations it is common to assume that boundary layer transition occurs at a discrete point rather than over a finite region. Furthermore, transition is assumed to occur when the Reynolds number (based on local conditions at the outer edge of the boundary layer and the distance from the leading edge of the plate) reaches some prescribed value. This prescribed transition Reynolds number is determined experimentally by noting the onset of transition and computing the Reynolds number at that point from known wind tunnel conditions. Unfortunately, however, transition is dependent upon other factors such as surface roughness, wind tunnel turbulence, and wall temperature in addition to the local Reynolds number. The practical significance of these additional dependencies is that one cannot be certain the transition will occur at the same location (i.e., distance from the leading edge) on two different flat plates even though the local Reynolds number is identical in both cases.

One is always hesitant to recommend a transition Reynolds number to use because of the uncertainties just described. Nevertheless, to make the graphs in this report

more useful, a flat plate transition Reynolds number of 1.5 x 10^6 is suggested. This commonly used value is subject to the shortcomings described above. It should be viewed as a guide rather than a strict criterion.

Figure 1 has been generated to enable the users of this report to determine whether a set of conditions chosen are above or below the transition Reynolds number. The curve is entered with a Mach number and altitude. If the point described by these two variables falls below the line representing the desired distance from the leading edge, the local Reynolds number exceeds 1.5 x 10^6 and a turbulent boundary layer is assumed to be present. Conversely, if the point falls above the line, a laminar boundary layer is assumed to exist.

The Johns Hopkins University
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Silver Spring, Maryland

PROCEDURE FOR USING CURVES

Assume it is desired to determine the equilibrium temperatures at points 1 and 6 feet back along the surface of a vehicle moving with a speed of Mach 8 at 100,000 feet. In order to determine which set of curves to use, refer immediately to Fig. 1. The point described by a Mach number of 8 and an altitude of 100,000 feet can be quickly determined. Since the point falls above the line representing x = 1 foot, the boundary layer is laminar 1 foot from the leading edge and Fig. 14 should be used to determine the equilibrium temperatures. However, since that same point lies below the line described by x = 6 feet, a turbulent boundary layer exists there. Therefore, Fig. 25 should be used to determine the equilibrium temperatures at x = 6 feet. (It is worth noting that for the Mach number and altitude ranges considered here, the flow is always turbulent if the distance from the beginning of boundary layer buildup is greater than approximately 4 feet.)

After the Mach number, altitude, and distance from the leading edge have been chosen, Figs. 14 and 25 still cannot be entered until a value has been chosen for the surface emissivity. If no value is known, it is suggested that the commonly used value of 0.8 be used. This is a realistic value since attempts are always made to keep the surface emissivity high in order to increase the radiation from the surface. The parameter η may now be obtained from Figs. 2 and 3. One foot behind the leading edge, where laminar flow is present, Fig. 2 yields a value of η equal to

0.8. On the other hand six feet behind the leading edge where turbulent flow is present, Fig. 3 is used to obtain a value of 1.15 for η. For the two values of η, Figs. 14 and 25 yield equilibrium temperatures of 1080°F and 1?20°F at locations 1 and 6 feet back of the leading edge, respectively.

From the same two graphs, values of the stagnation temperature and adiabatic wall temperatures can also be obtained. Figure 25 will yield a stagnation temperature of 4150°F which may be used as a conservative estimate of the stagnation point wall temperature. Laminar and turbulent values of the adiabatic wall temperature, 3680°F and 3920°F respectively, can be obtained from Figs. 14 and 25, respectively. If no radiation cooling were present, the temperature of perfectly insulated surfaces would rise to these adiabatic wall temperatures.

CONCLUDING REMARKS.

Flat plate equilibrium temperatures have been computed as functions of Mach number, altitude, distance from the leading edge, and surface emissivity. In addition, stagnation and adiabatic wall temperatures have been determined as functions of Mach number and altitude up to a Mach number of 20 and an altitude of 100,000 feet. The results have been presented in convenient graphical form comprising two sets of curves; one for a laminar boundary layer and the other for a turbulent boundary layer. In addition, a curve is presented based on a transition Reynolds number of 1.5×10^6, whereby, the user of the graphs can readily determine whether, for a given set of conditions, the laminar or turbulent boundary layer curves should be used. It is expected that these curves will be extremely useful during the preliminary design phase of high velocity vehicles.

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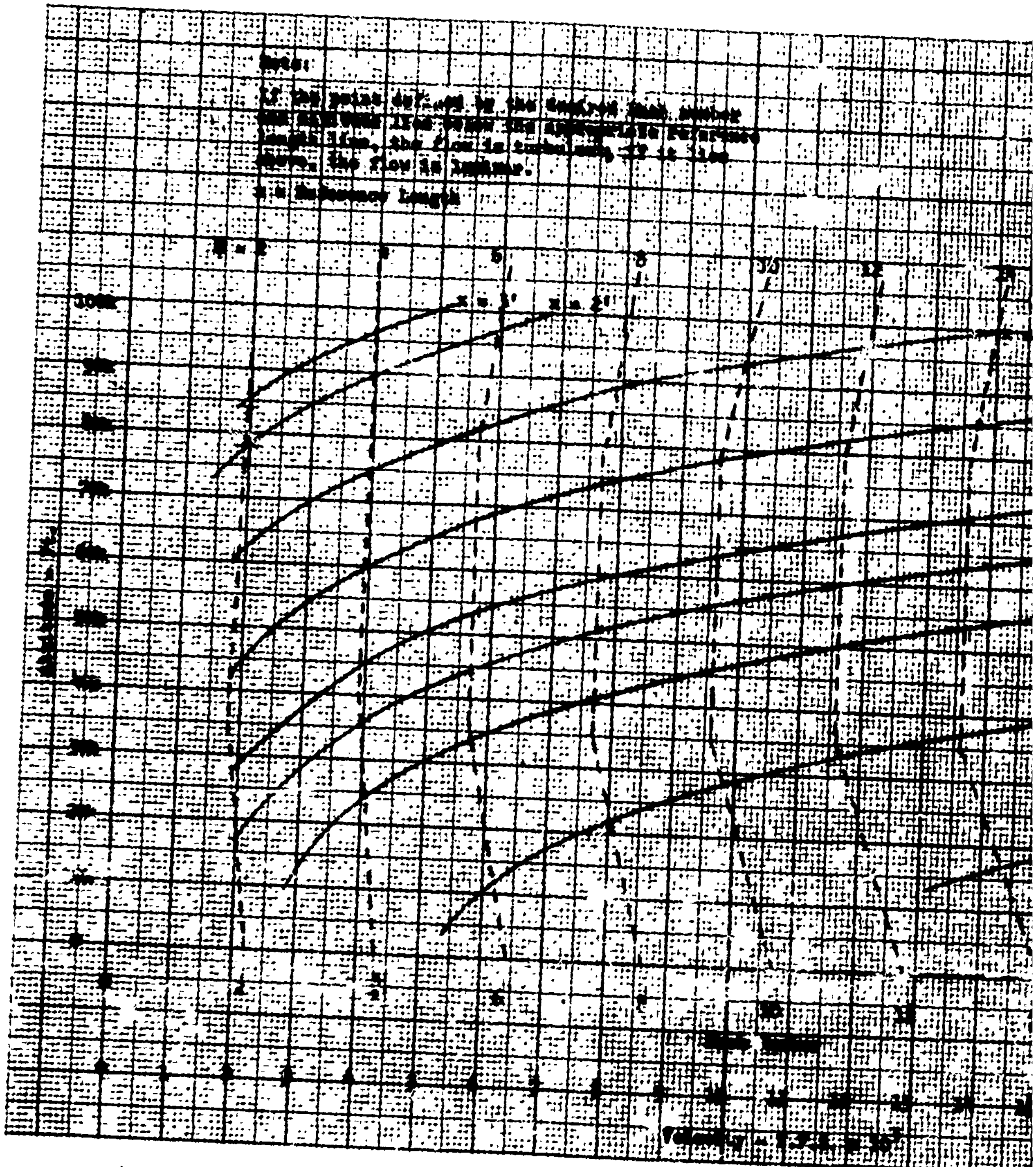


Figure 1 CHART FOR DETERMINING LAMINAR AND TURBULENT FLOW R

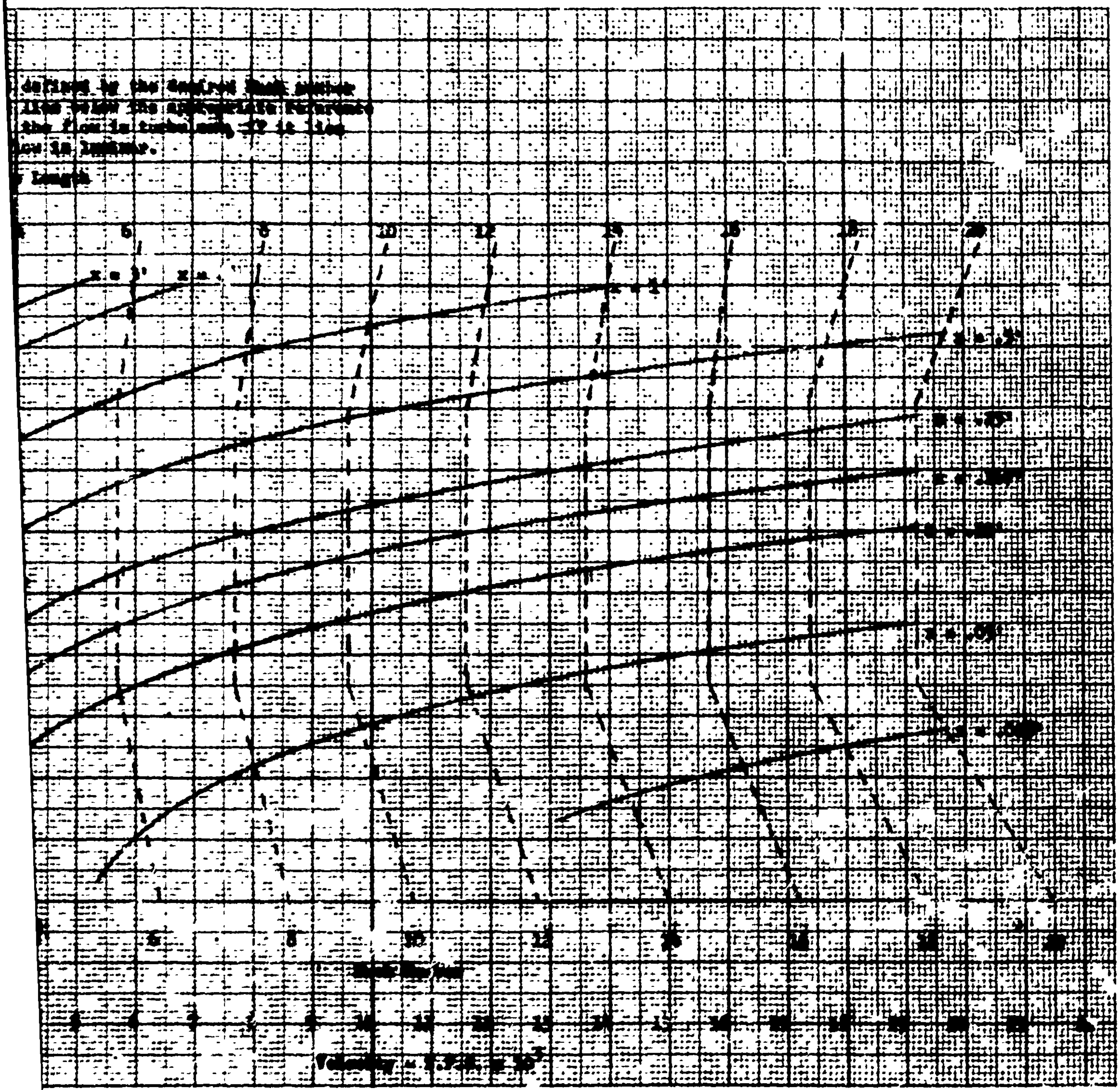
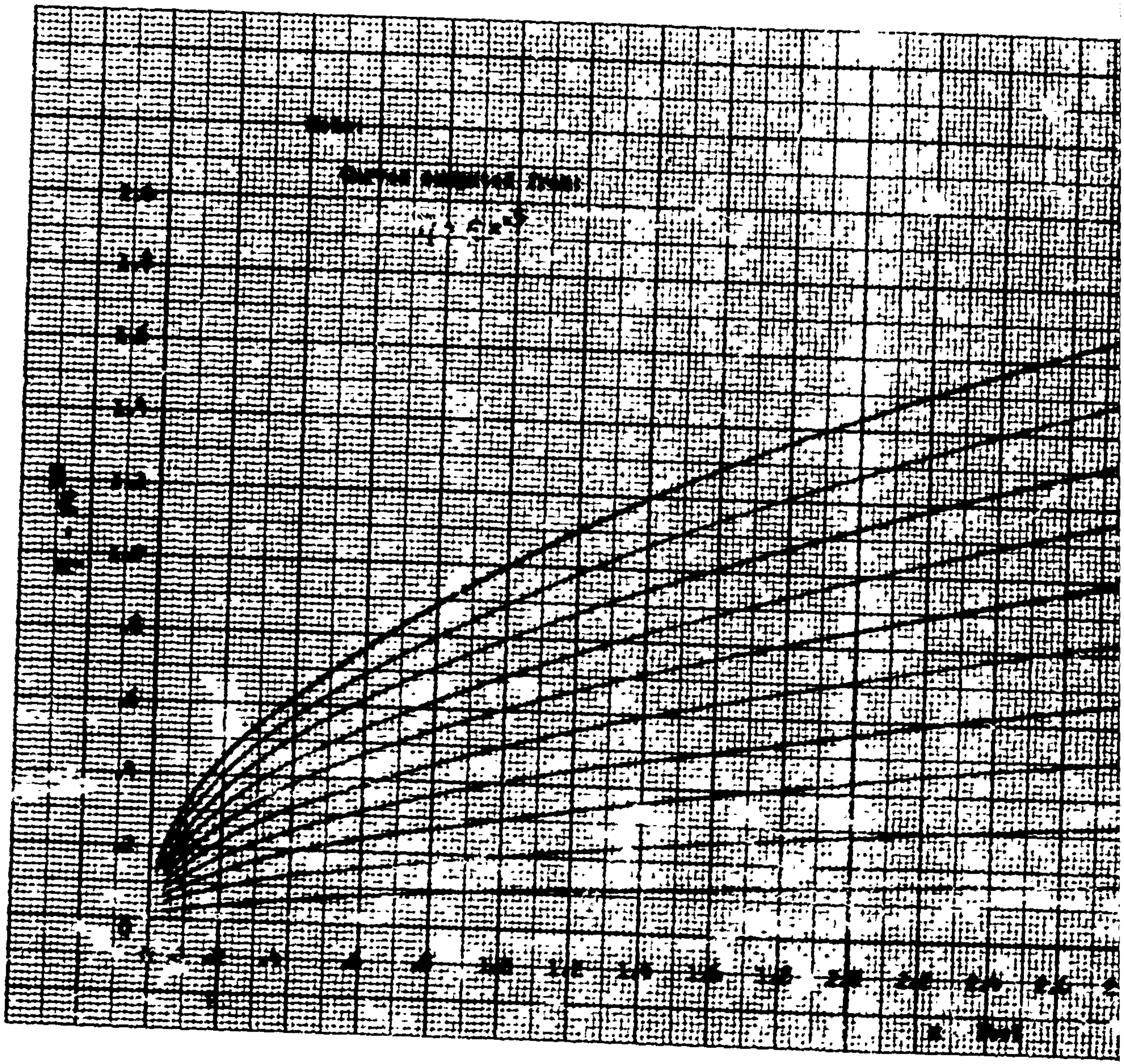


Figure 1 CHART FOR DETERMINING LAMINAR AND TURBULENT FLOW REGIMES



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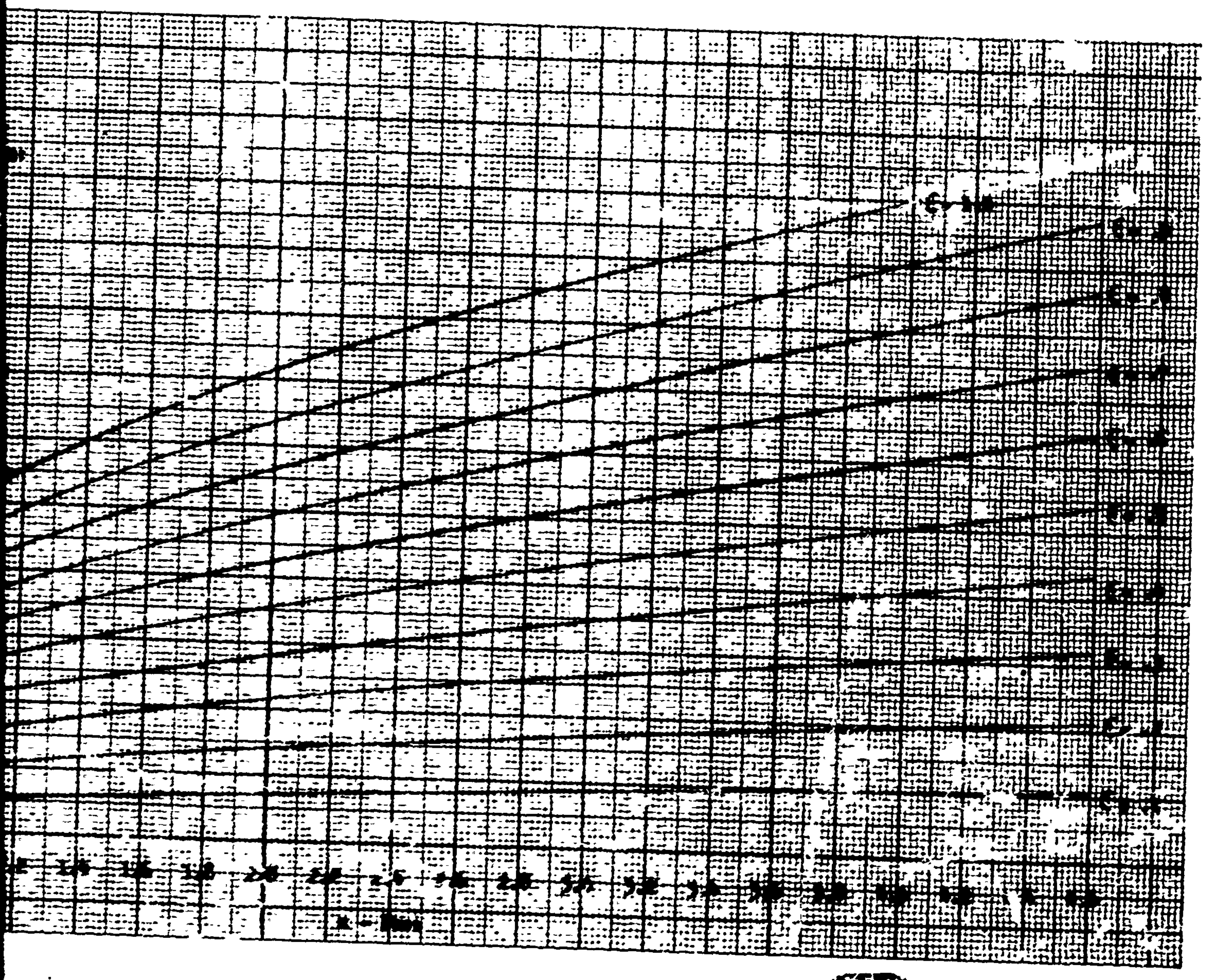

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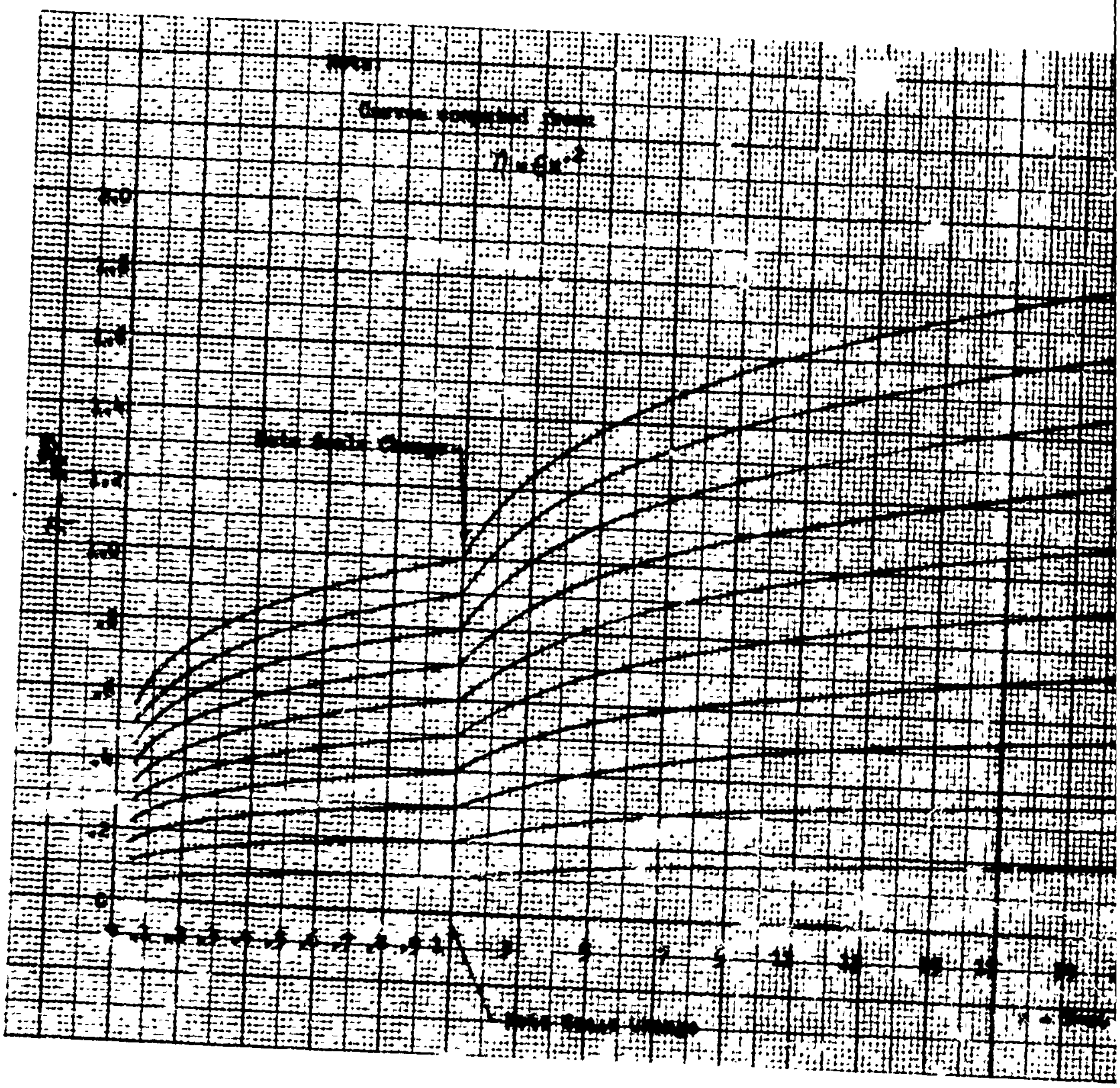
Figure 2 PARAMETER, η, VERSUS DISTANCE FROM BEGINNING OF BOUNDARY LAYER BUILD-UP - LAMINAR FLOW



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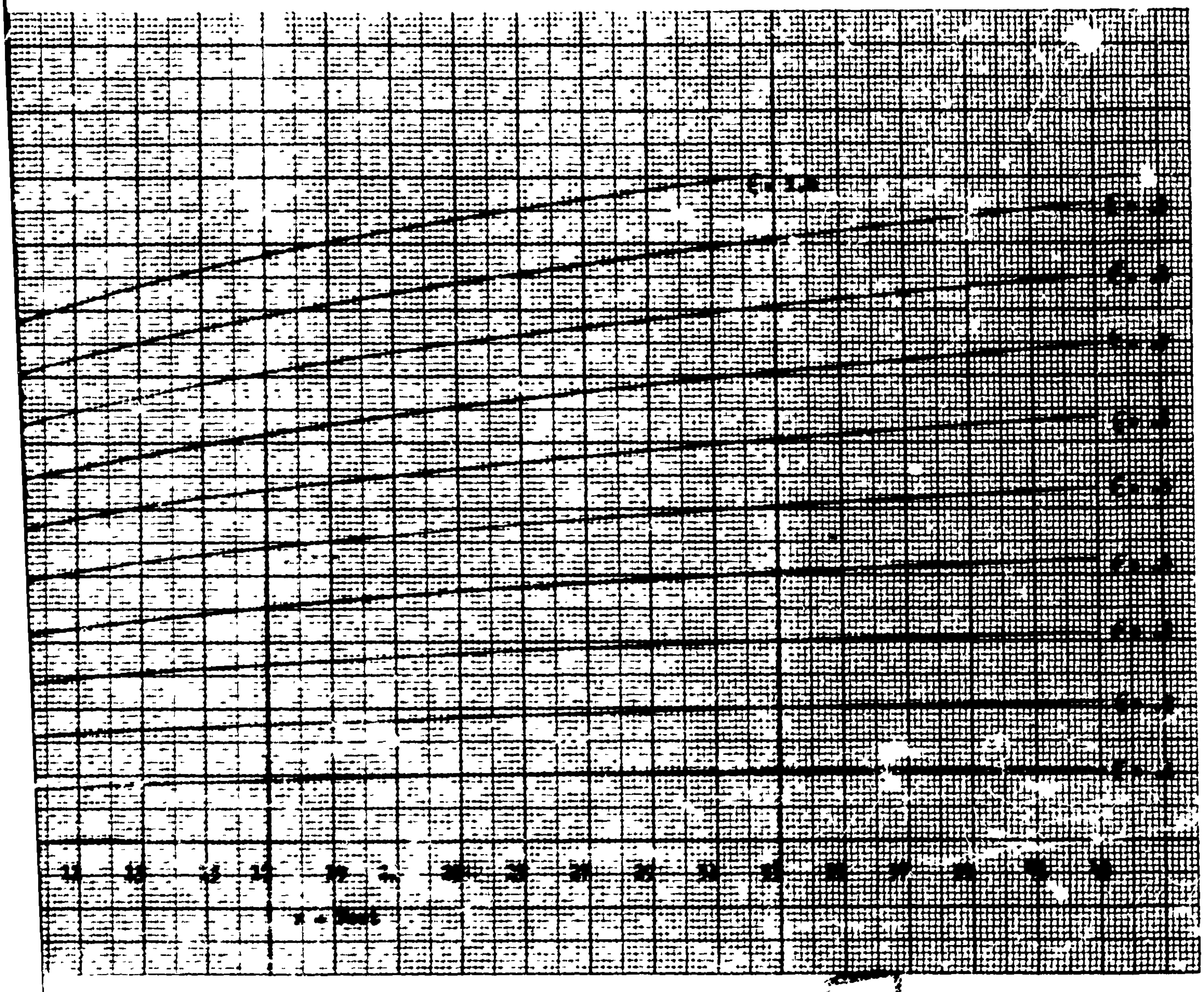

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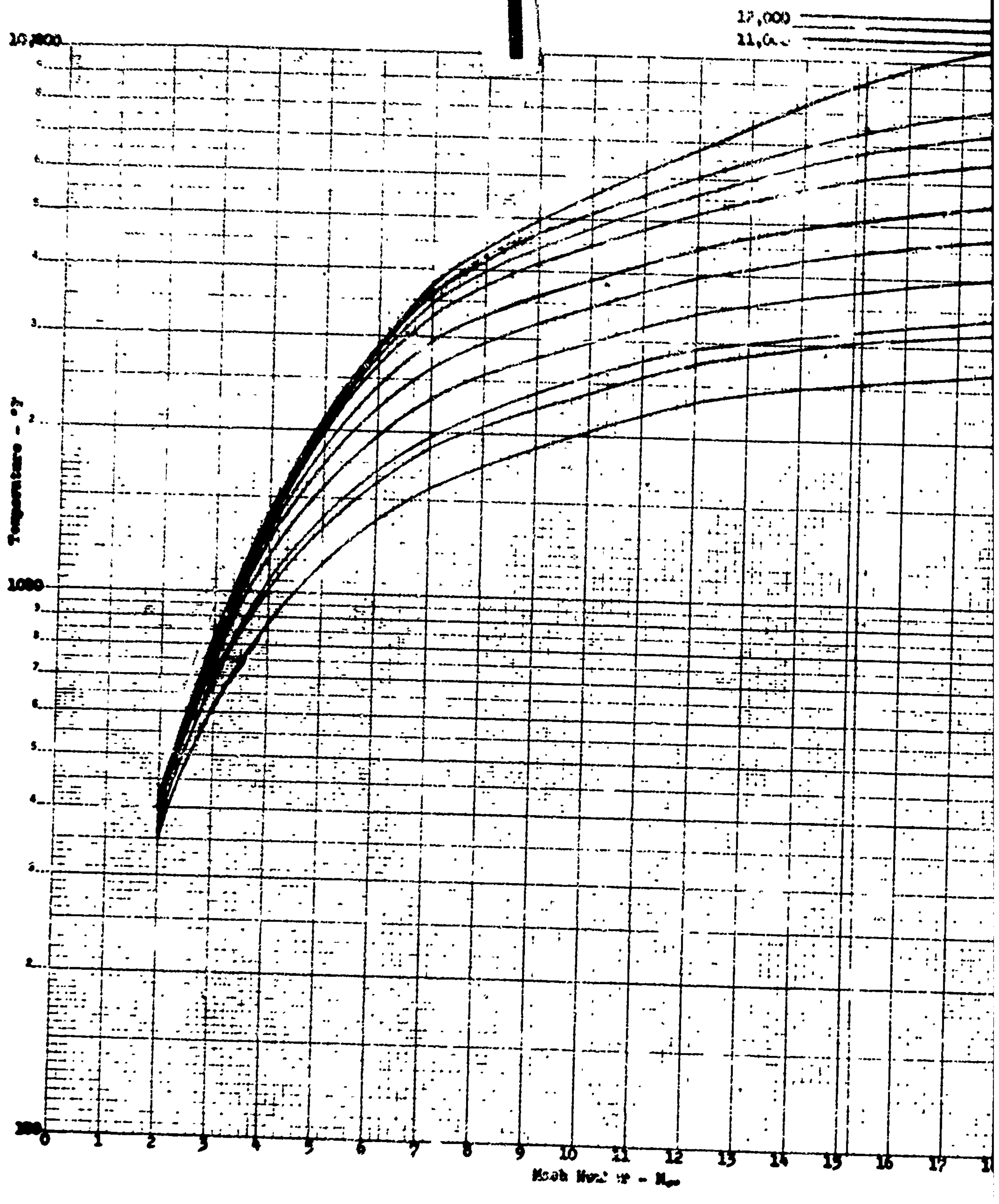
Figure 3 PARAMETER, ?, VERSUS DISTANCE FROM BEGINNING OF BOUNDARY LAYER BUILD-UP - TURBULENT FLOW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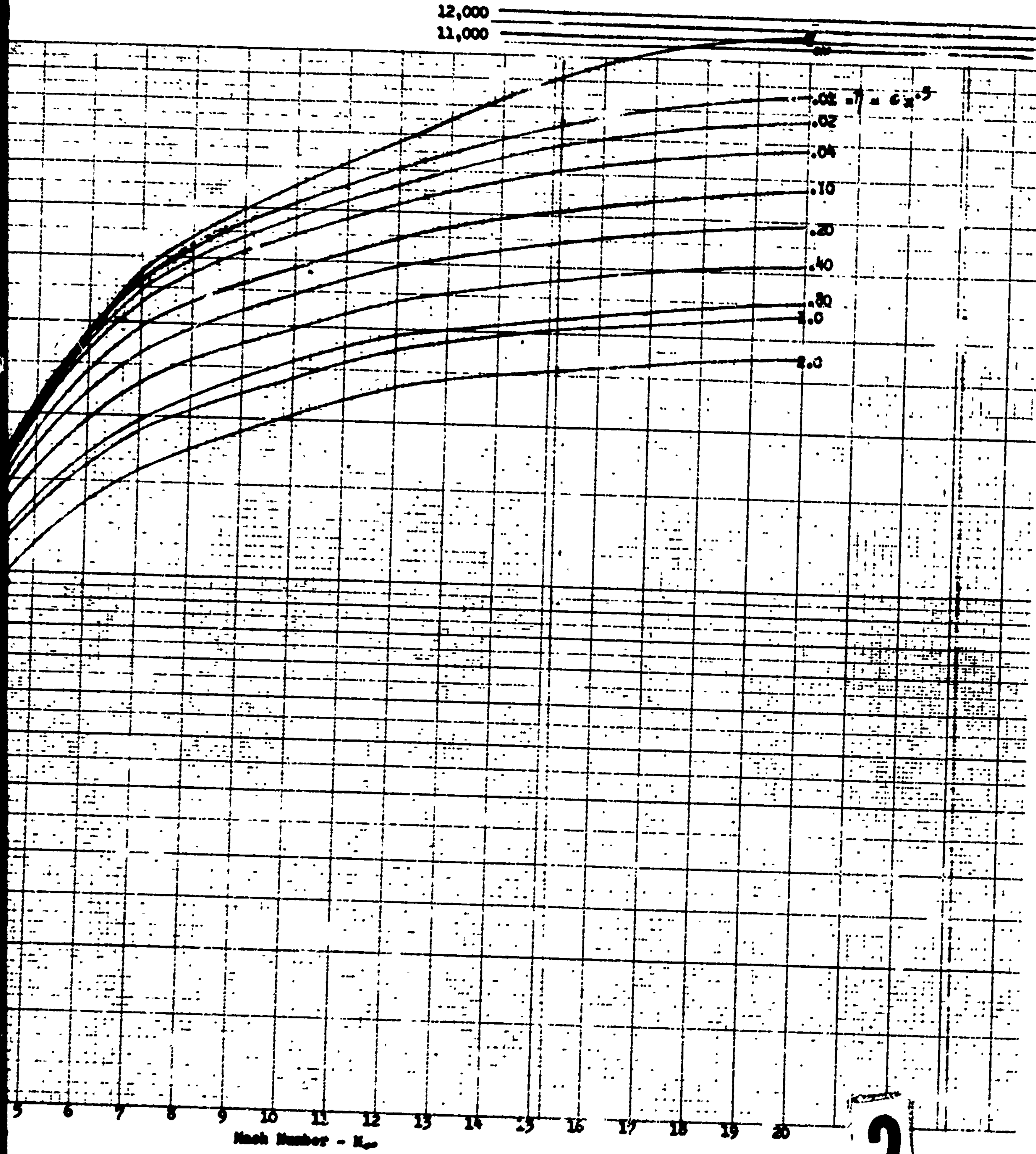

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Figure 4

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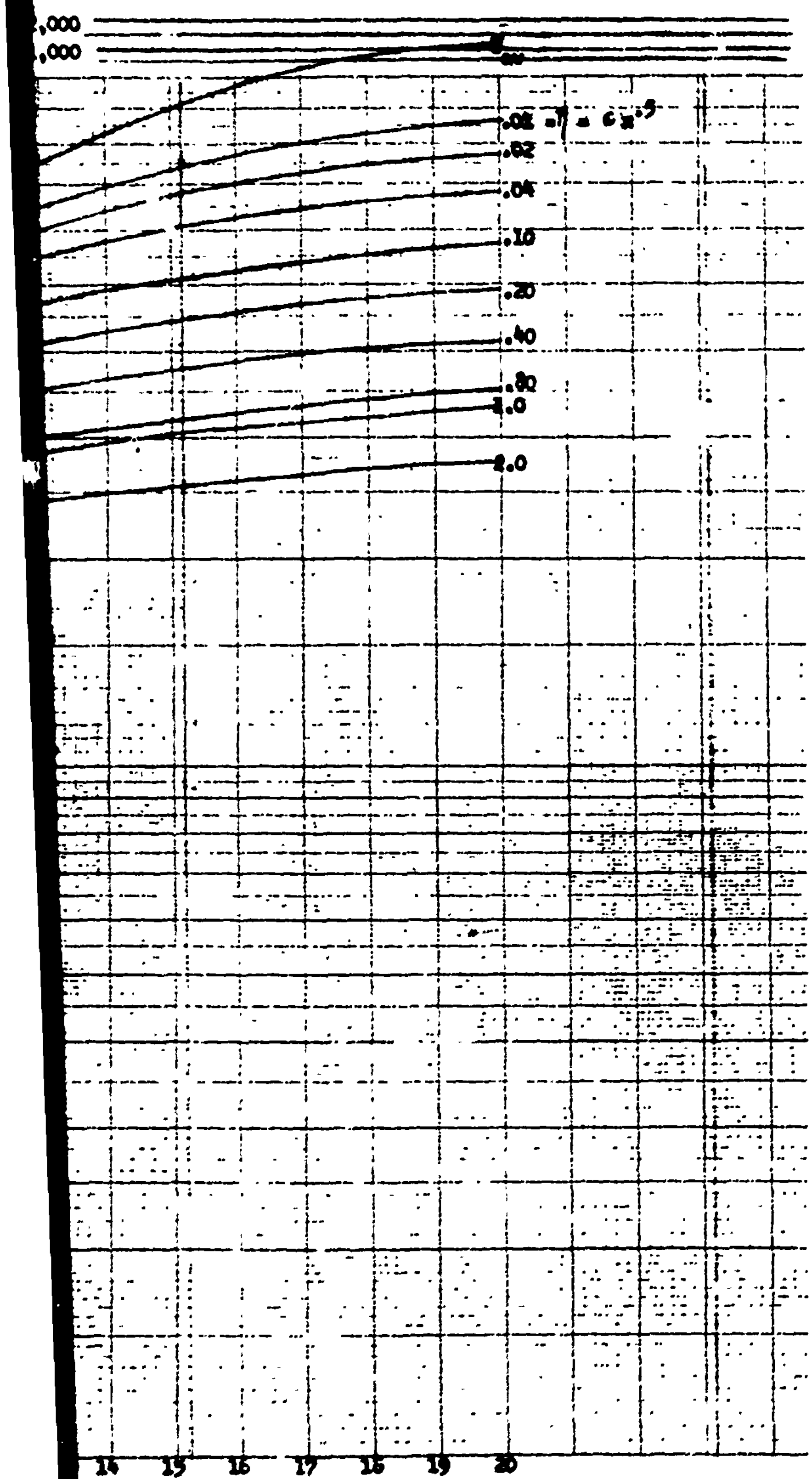

Figure 4 EQUILIBRIUM AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - LAMINAR BOUNDARY LAYER ALTITUDE = 0 Ft.

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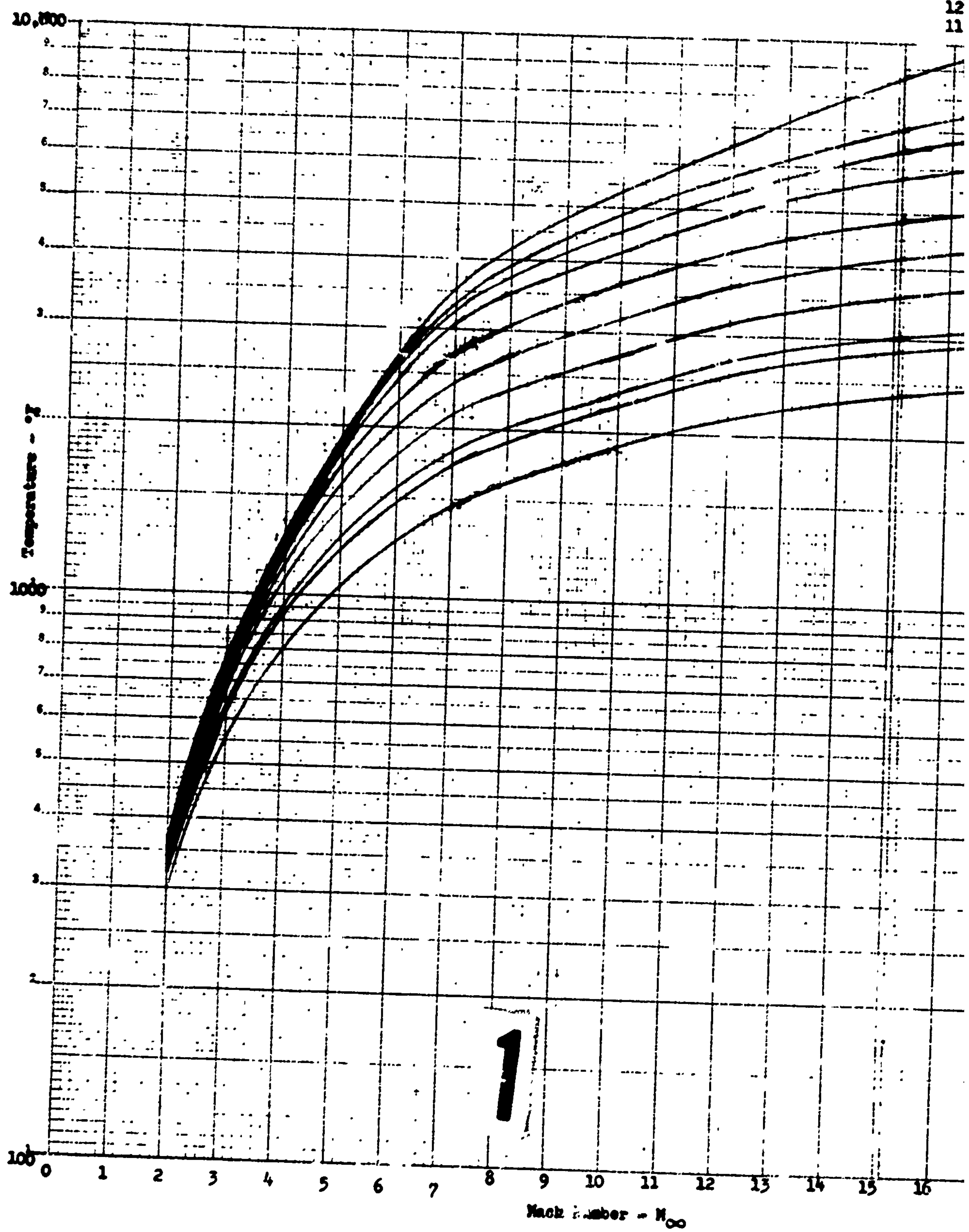
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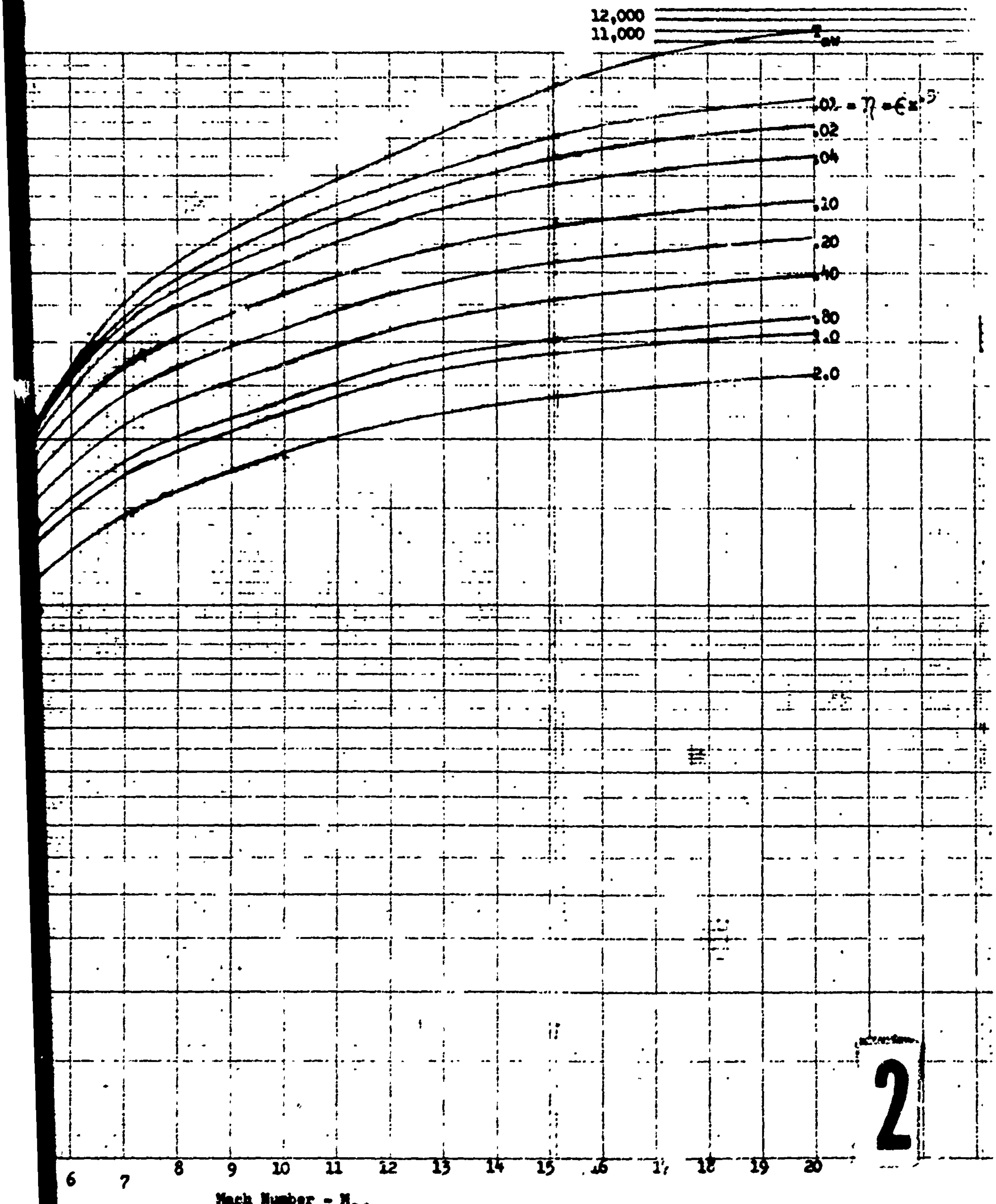

Figure 5 EQUILIBRI
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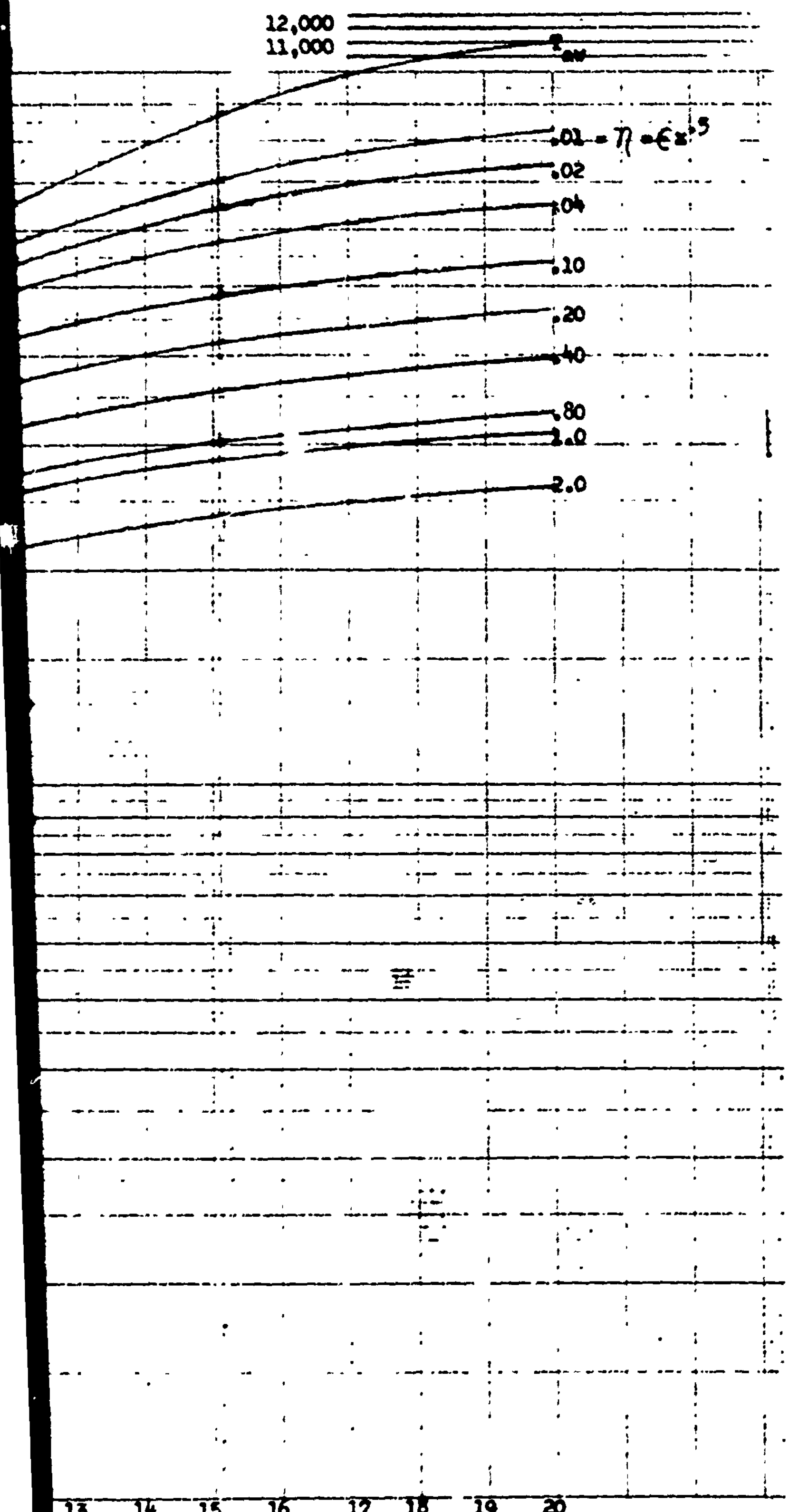
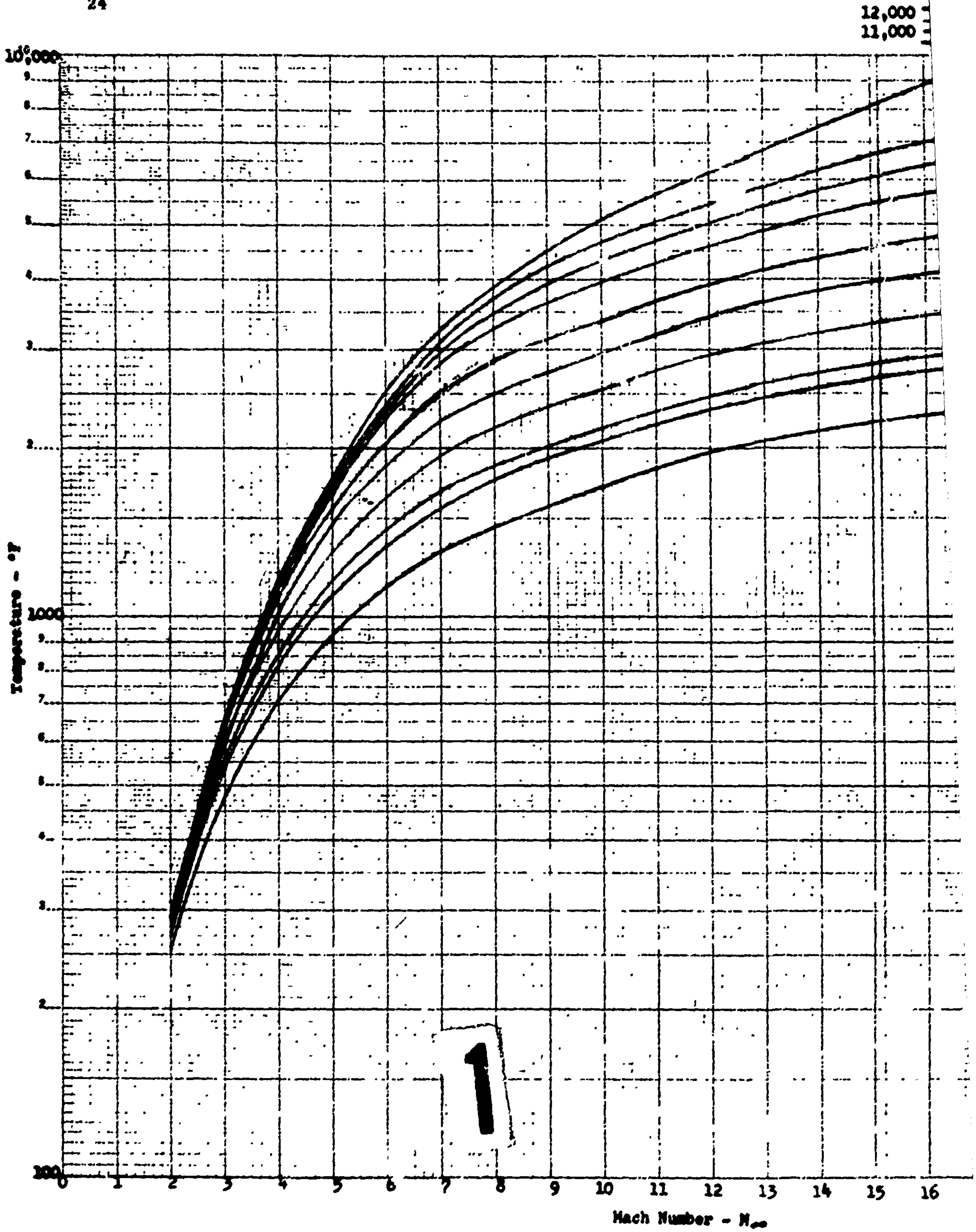


Figure 5 EQUILIBRIUM AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - LAMINAR BOUNDARY LAYER ALTITUDE = 10,000 FEET

3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Mach Number - M_∞
Temperature - °F
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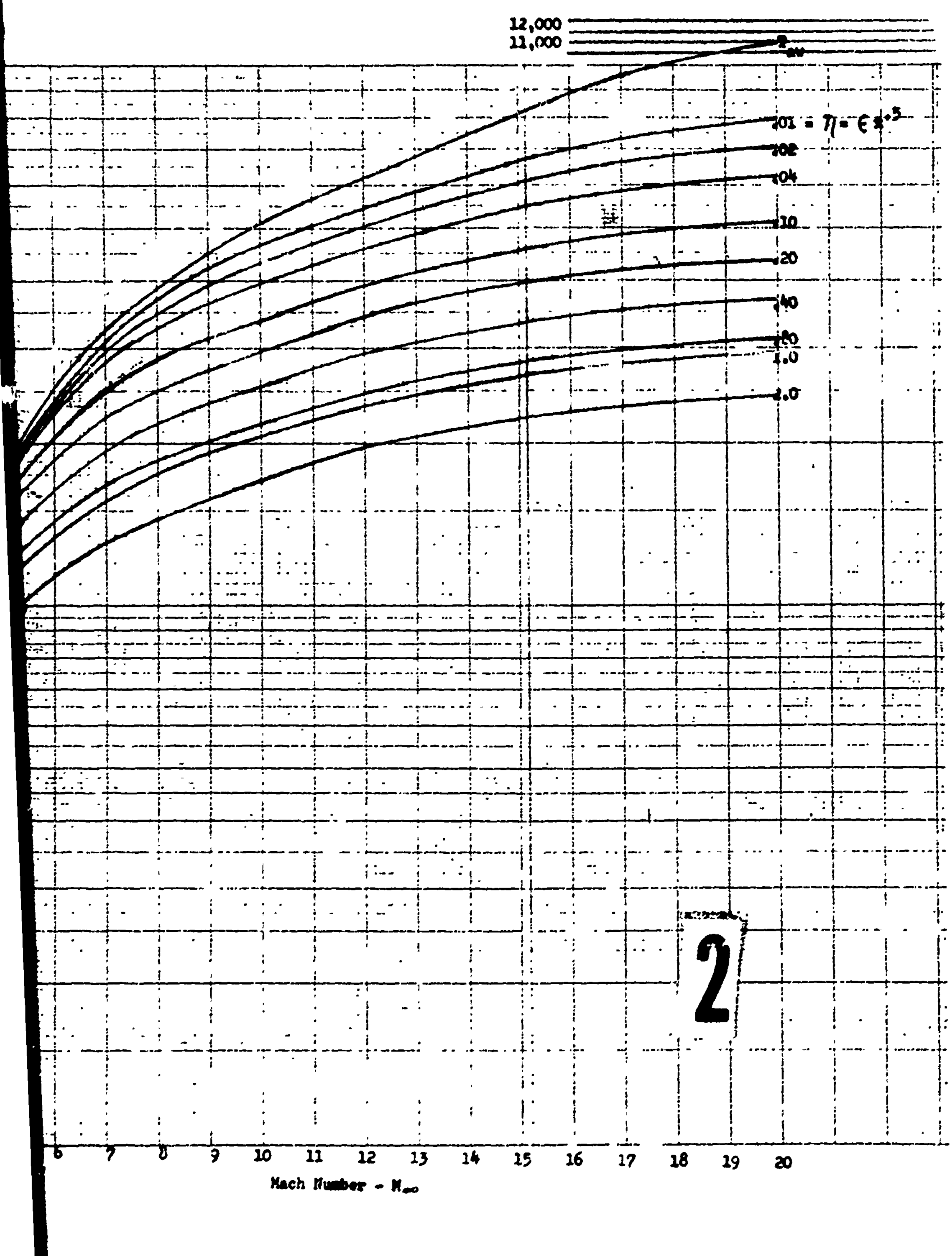

Figure 6 EQUILIBRI
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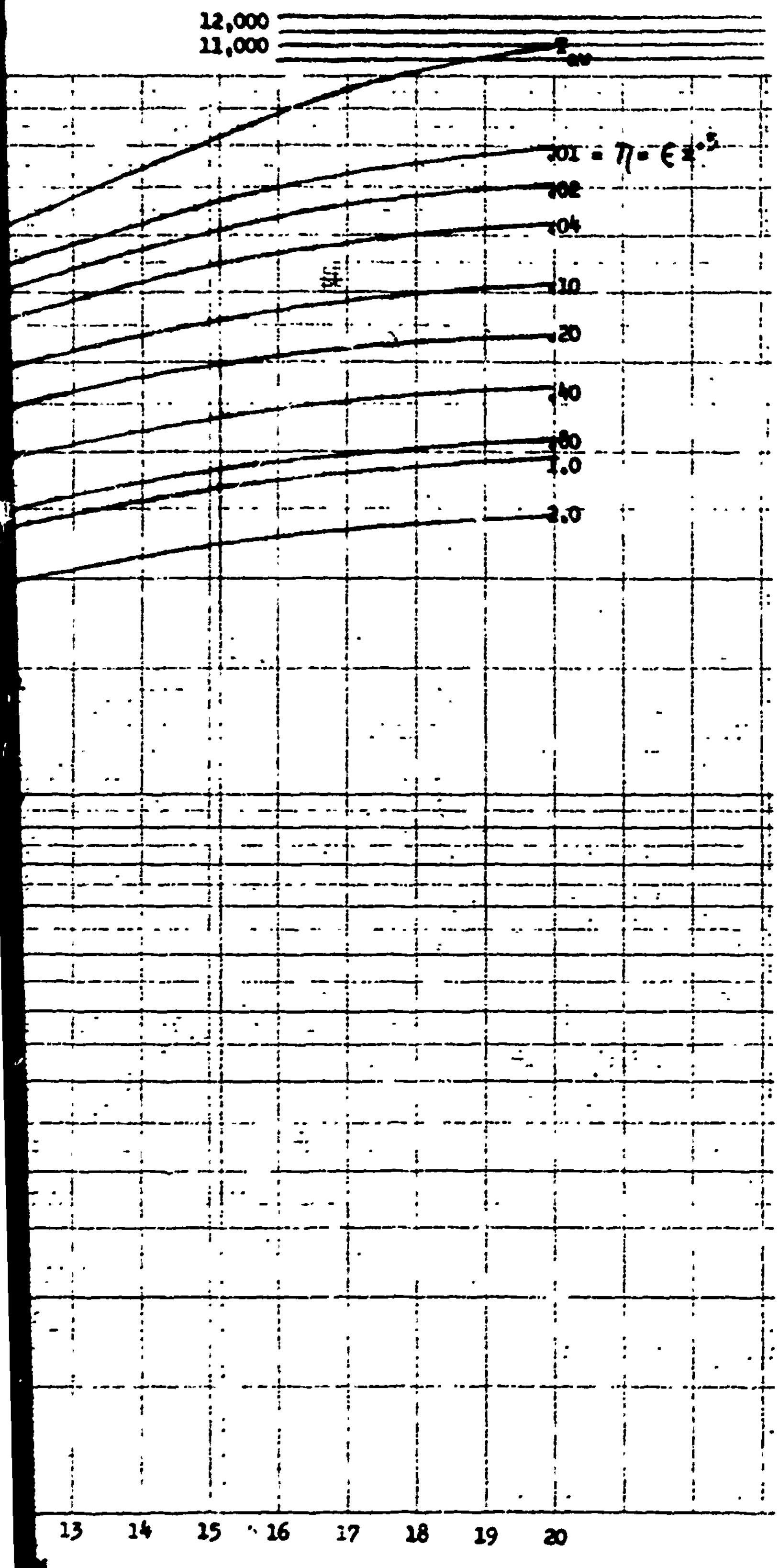

Figure 6 EQUILIBRIUM AND ADIABATIC WALL
TEMPERATURE VERSUS MACH NUMBER
STANDARD DAY - LAMINAR BOUNDARY LAYER
ALTITUDE = 20,000 FEET

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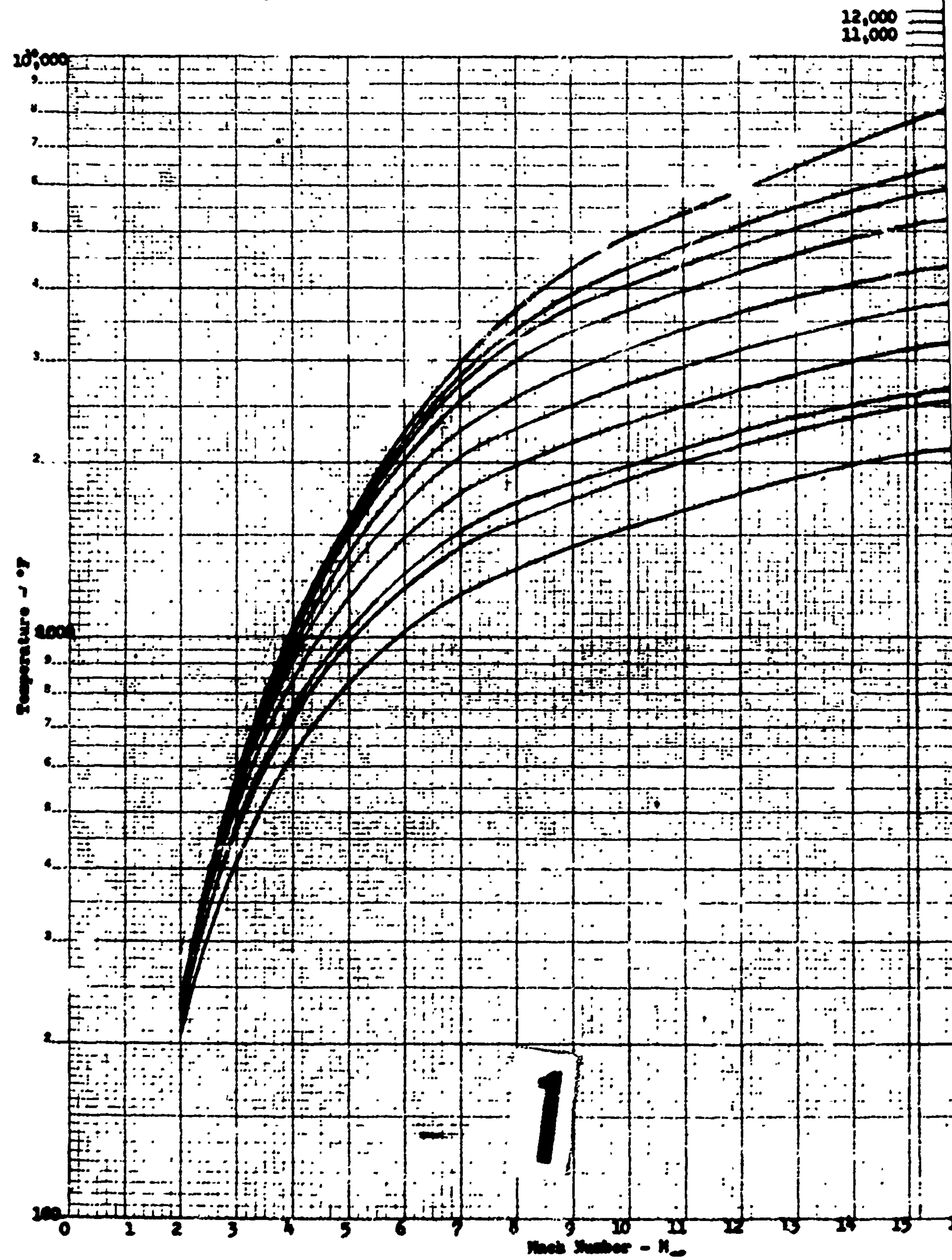
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Temperature - °F
Mach Number - M_∞
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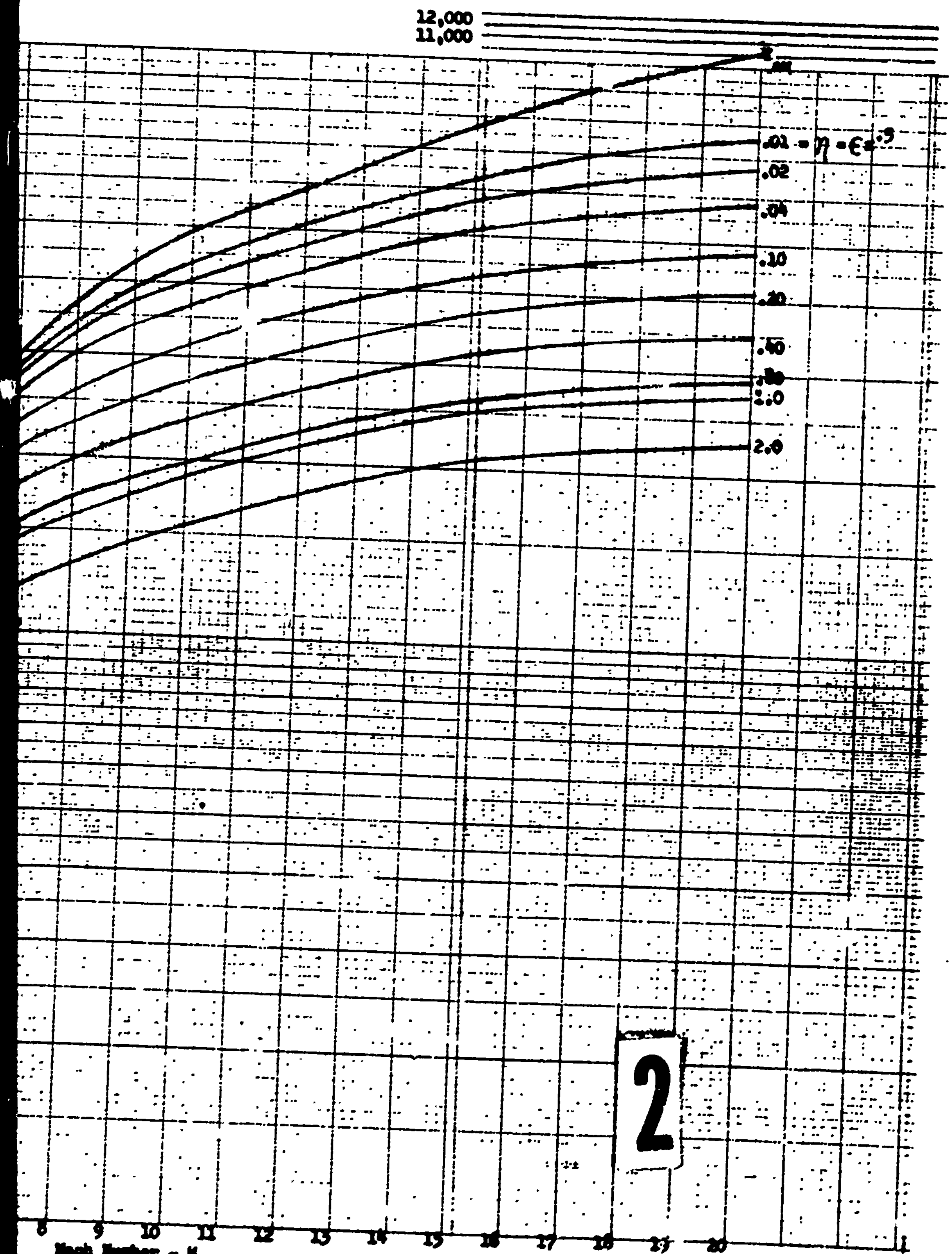

Figure 7 EQUILIBRIUM AND ADIA
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STANDARD DAY - LAMINAR
ALTITUDE = 30,0

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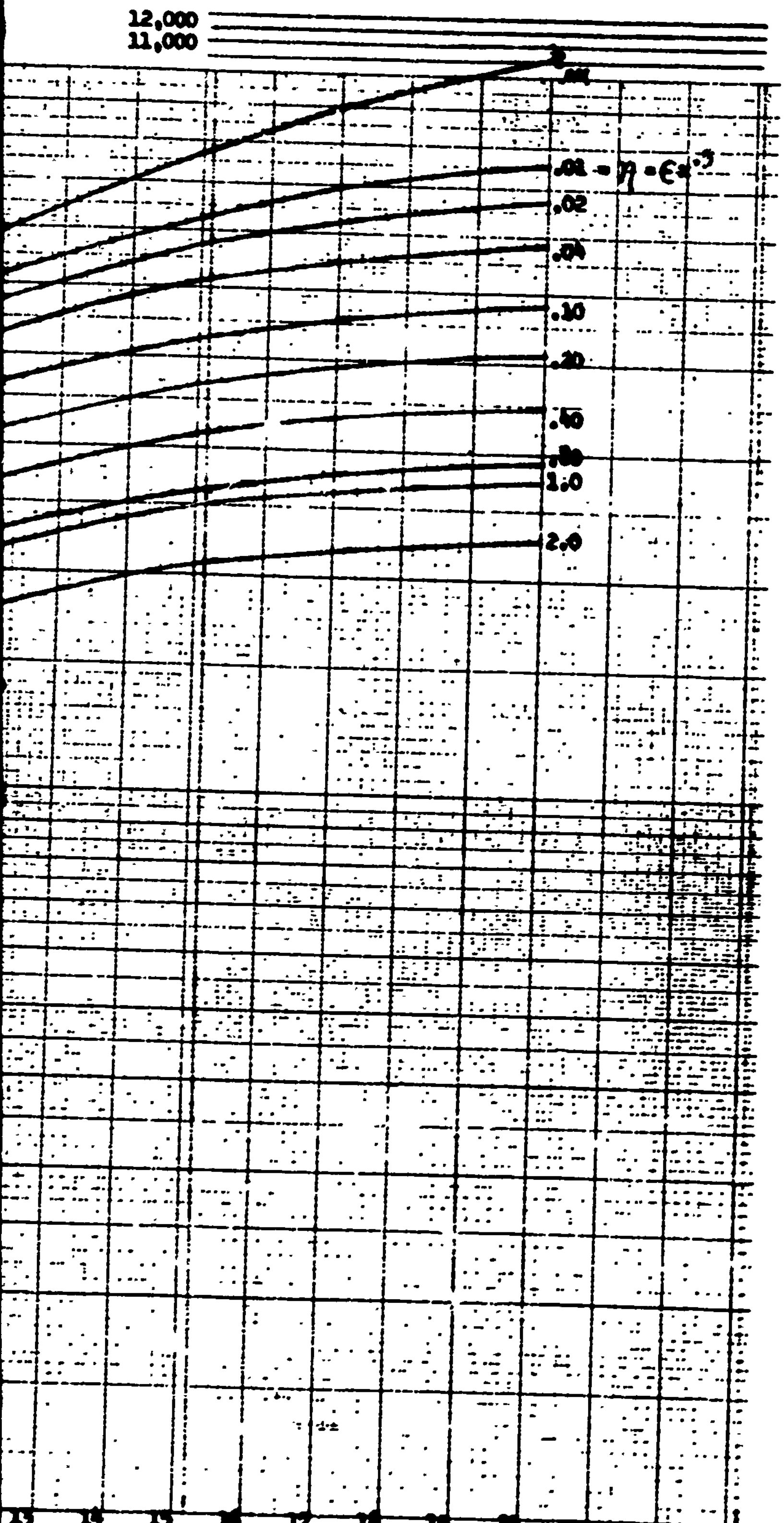
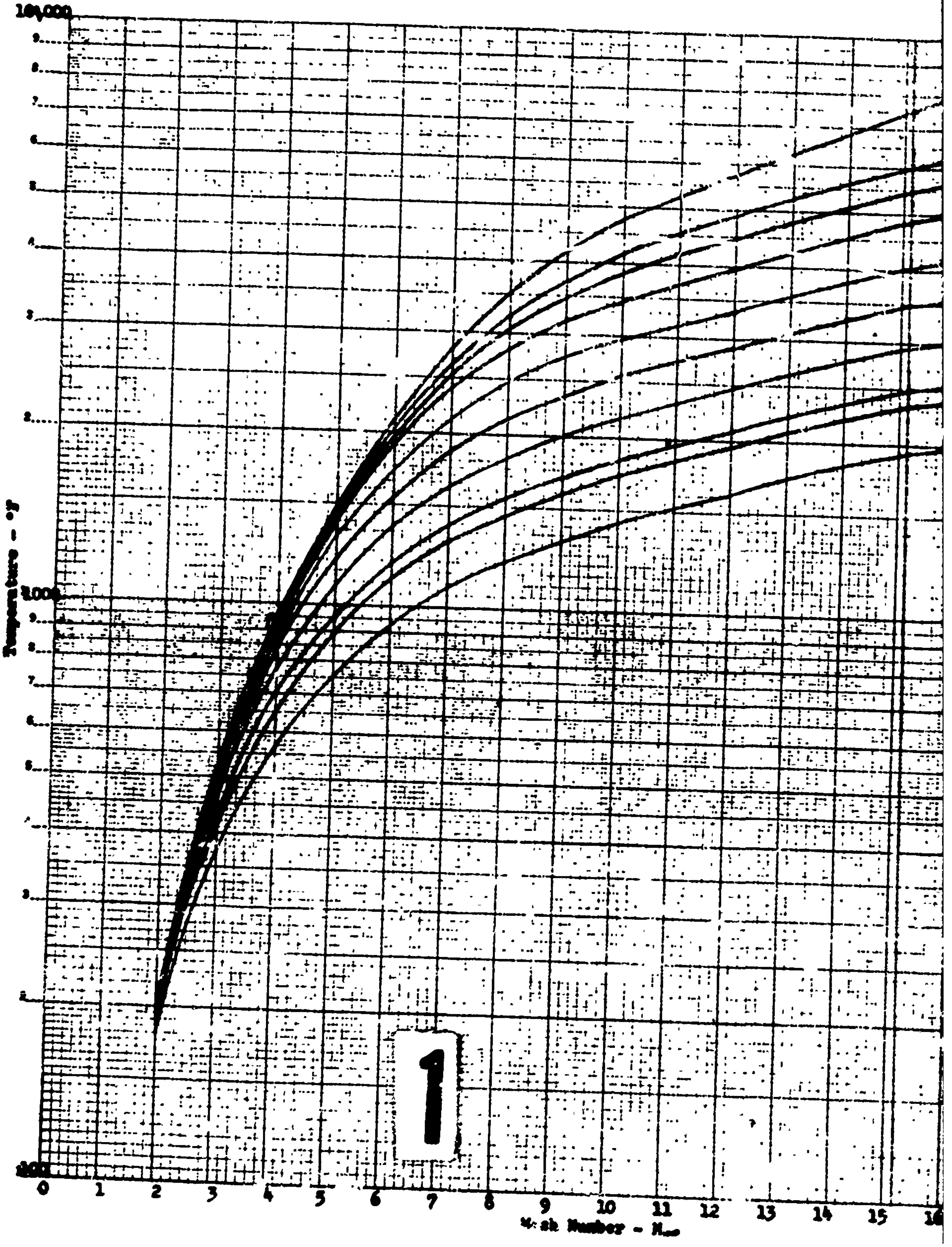


Figure 7 EQUILIBRIUM AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - LAMINAR BOUNDARY LAYER ALTITUDE = 30,000 FEET

3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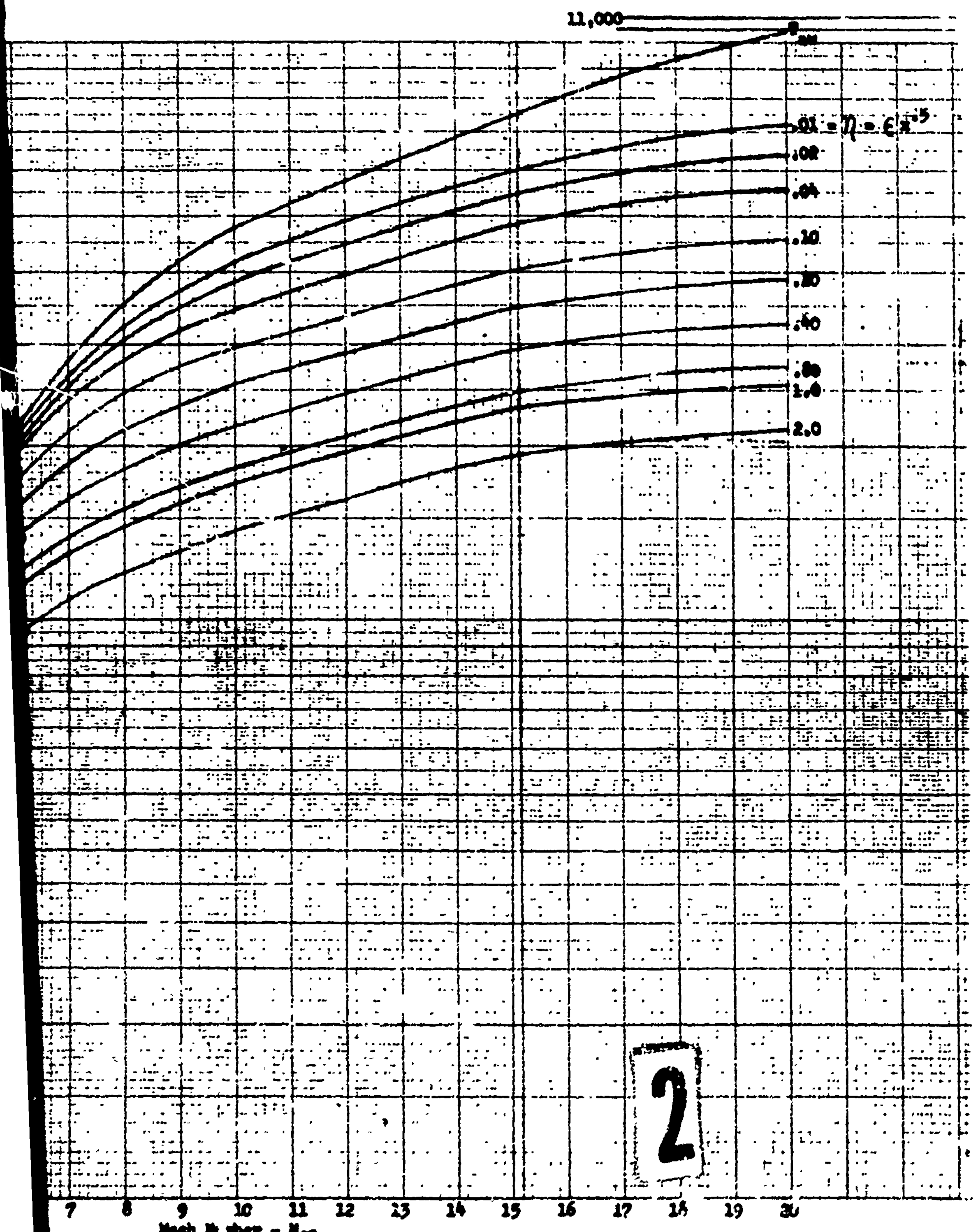

Figure 8 EQUILIBRIUM AM
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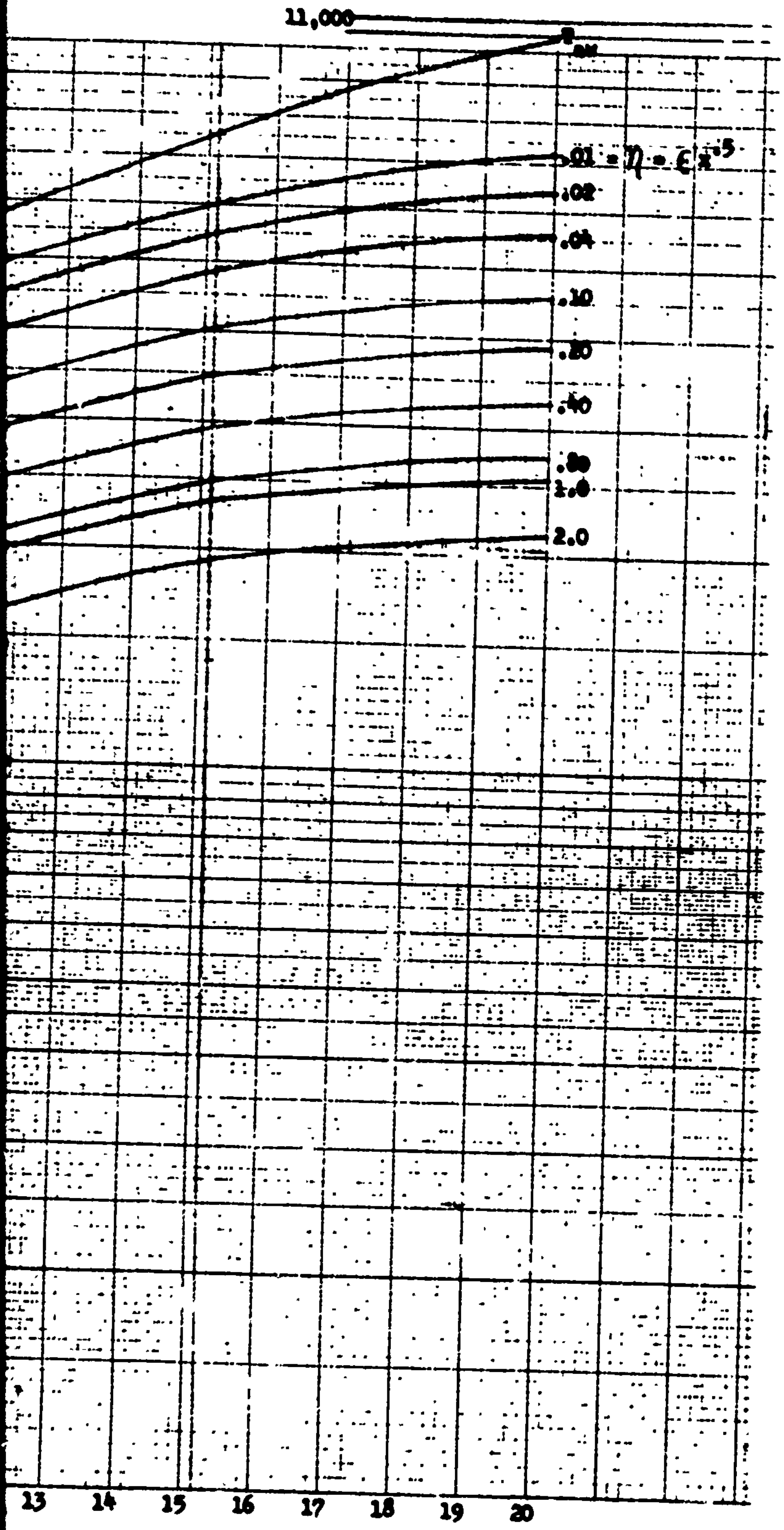
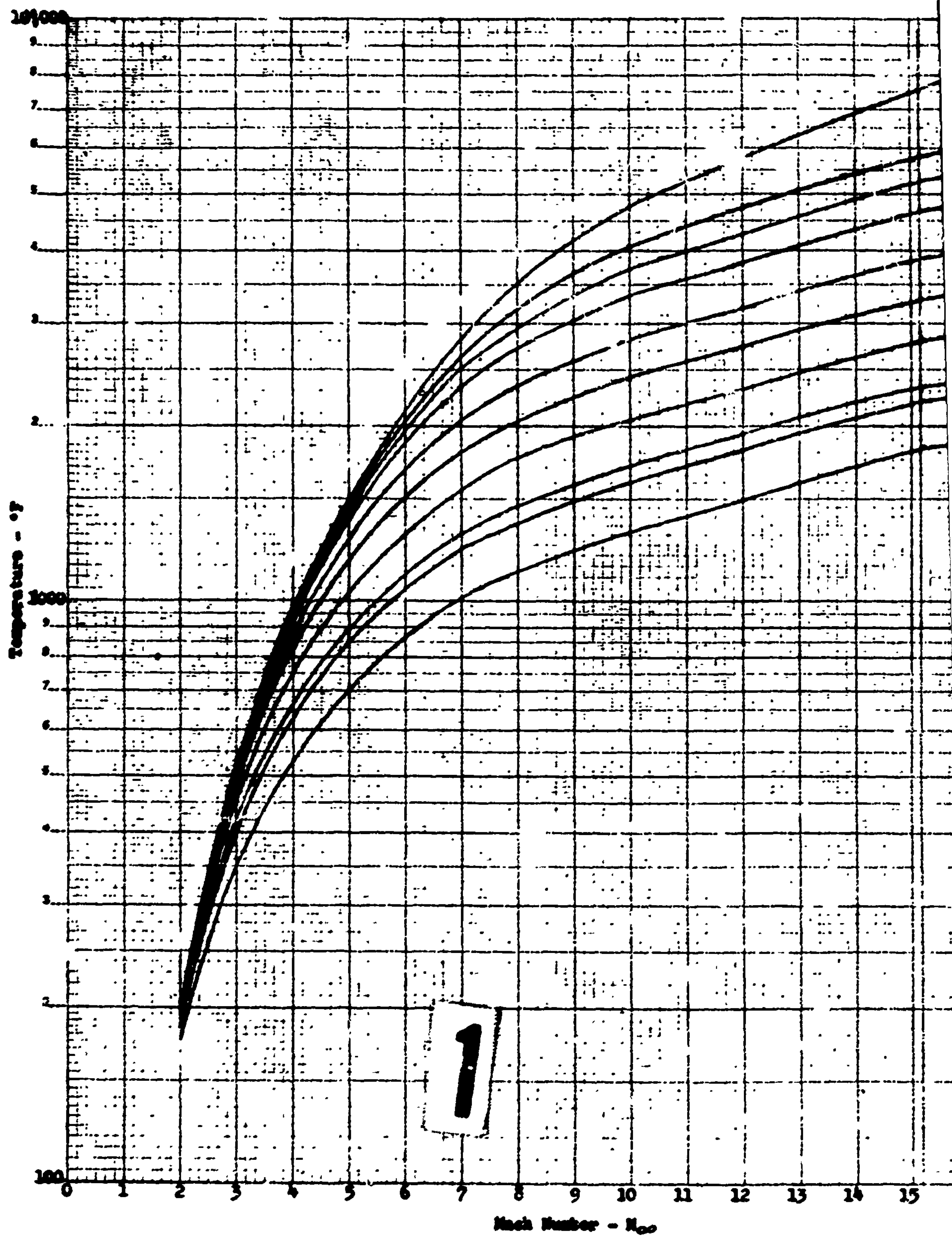


Figure 8 EQUILIBRIUM AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER
STANDARD DAY - LAMINAR BOUNDARY LAYER
ALTITUDE = 40,000 FEET

3



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Mach Number - M_∞
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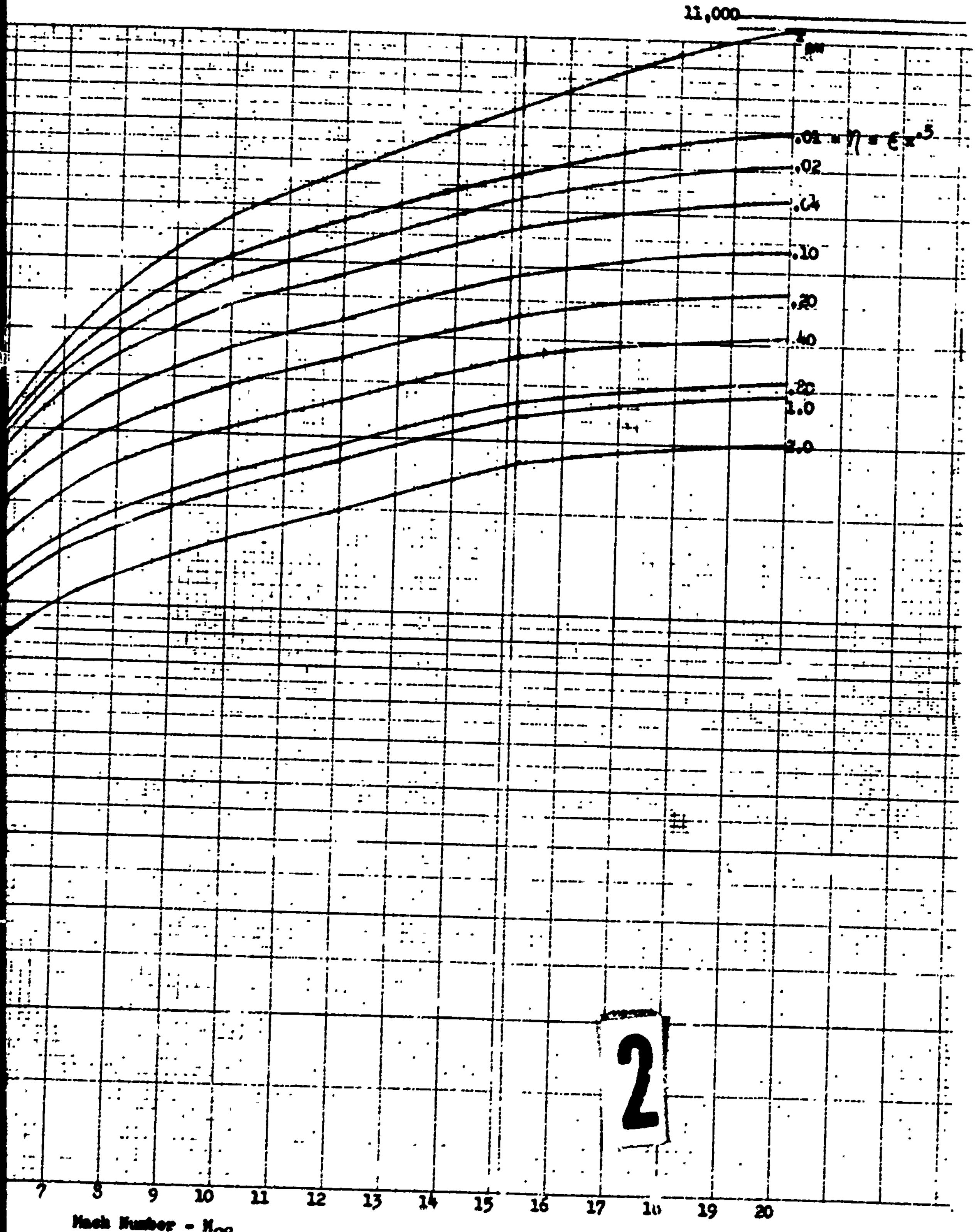

Figure 9 EQUILIBRIUM AN
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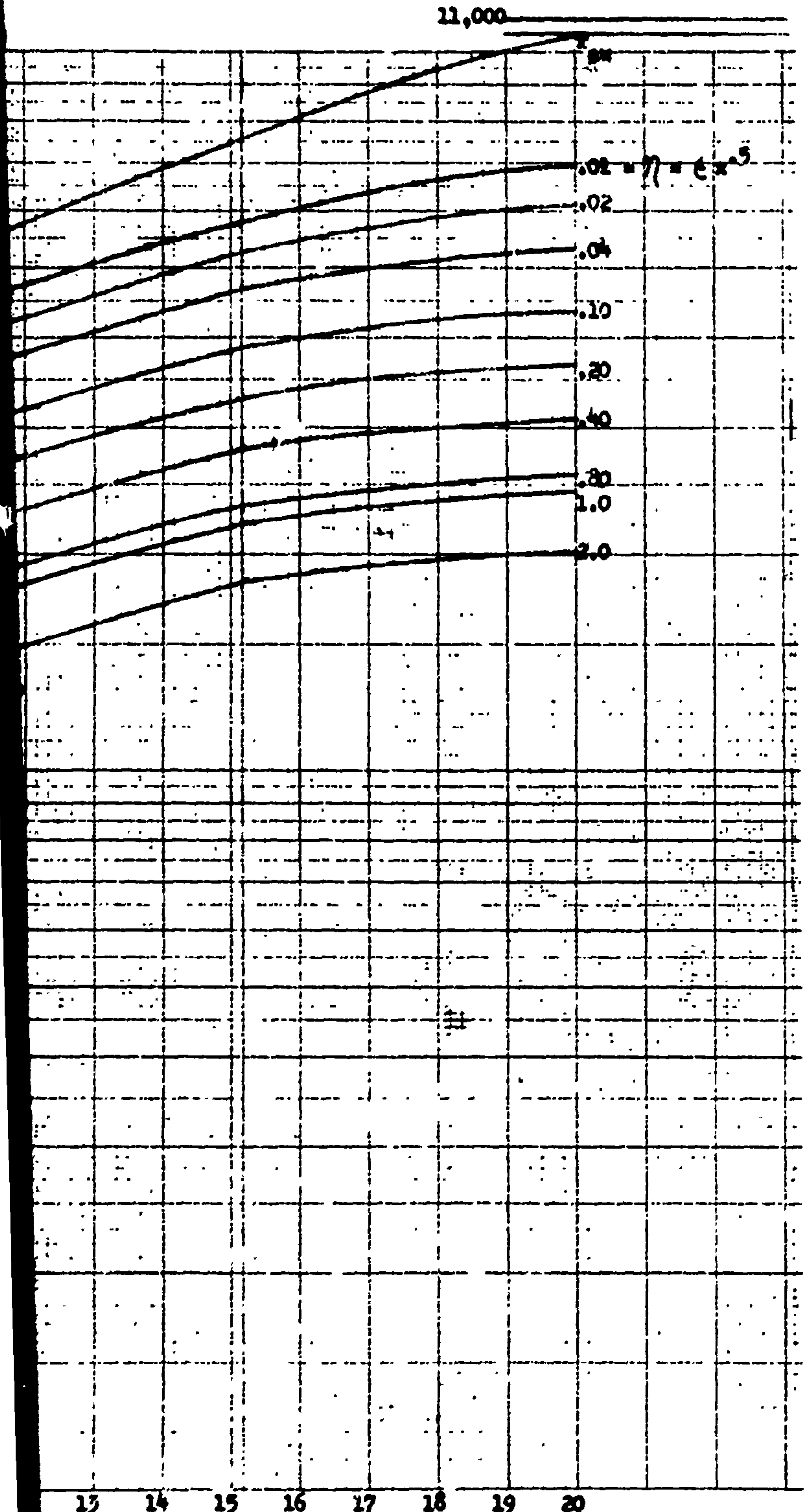
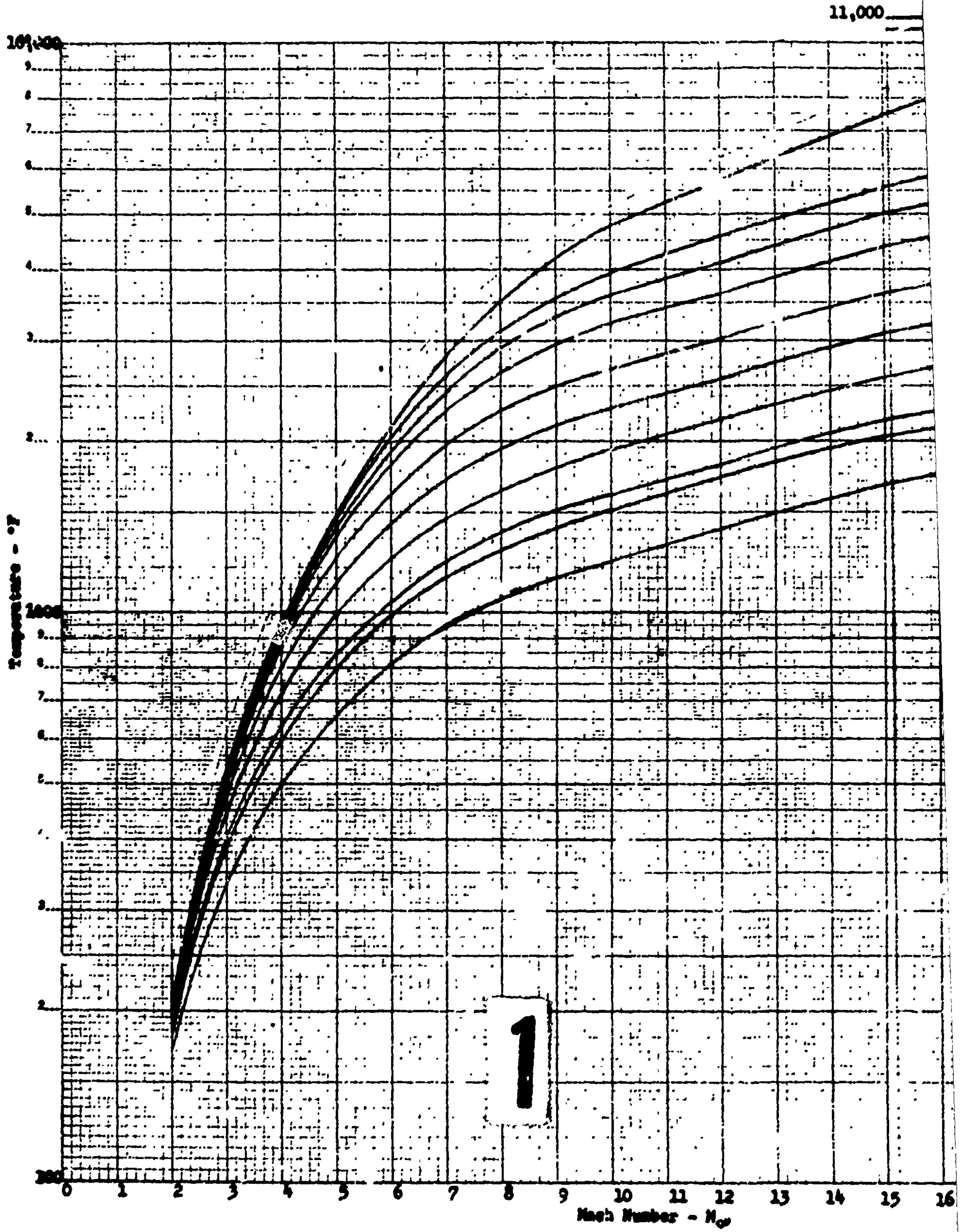


Figure 9 EQUILIBRIUM AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER
STANDARD DAY - LAMINAR BOUNDARY LAYER
ALTITUDE = 50,000 FEET

3


11,000
Temperature - °F
Mach Number - M_∞
1

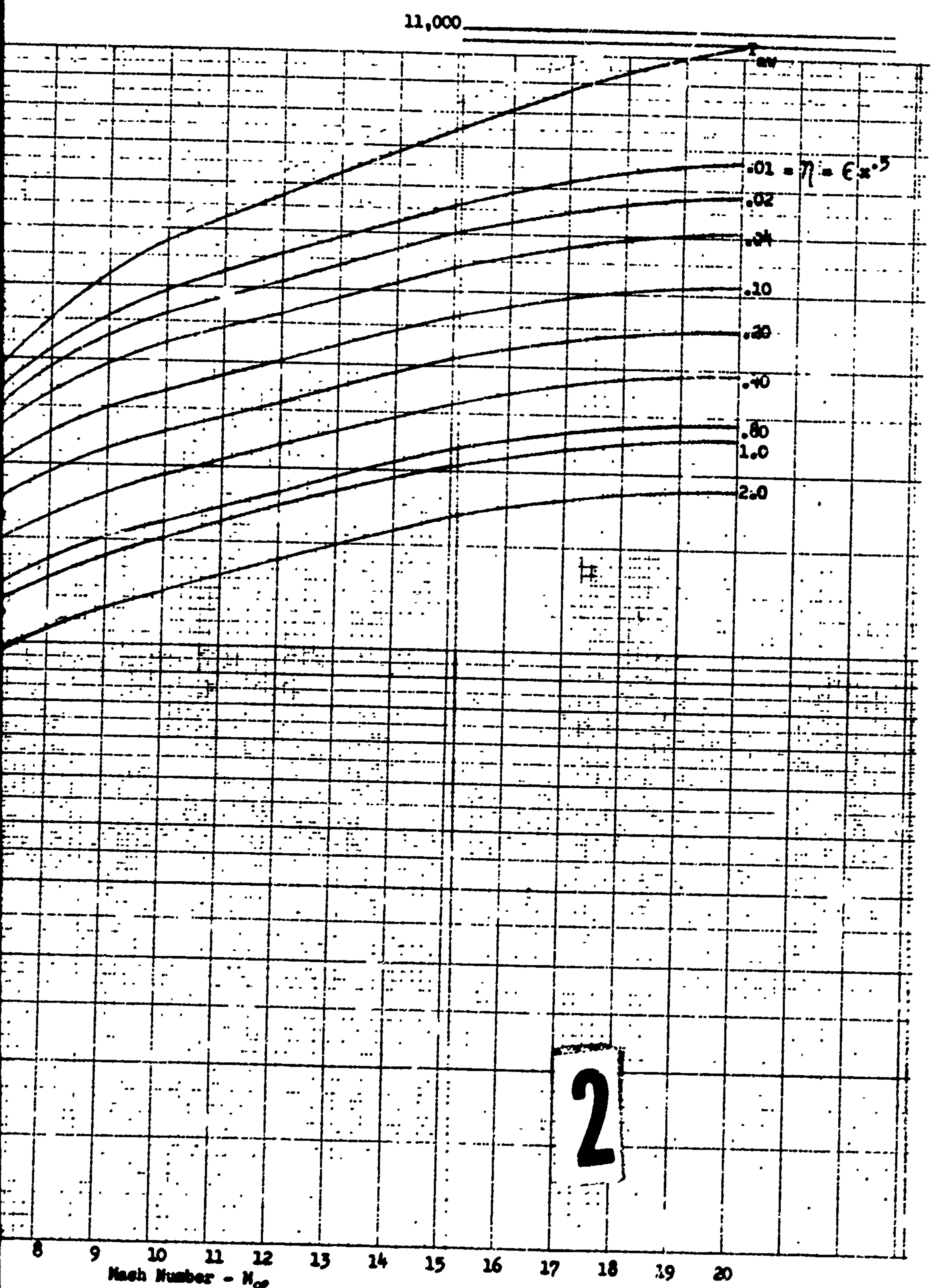


Figure 10 EQUILIBRIUM AND ADIA
TEMPERATURE VERSUS
STANDARD DAY - LAMINAR
ALTITUDE = 60,00

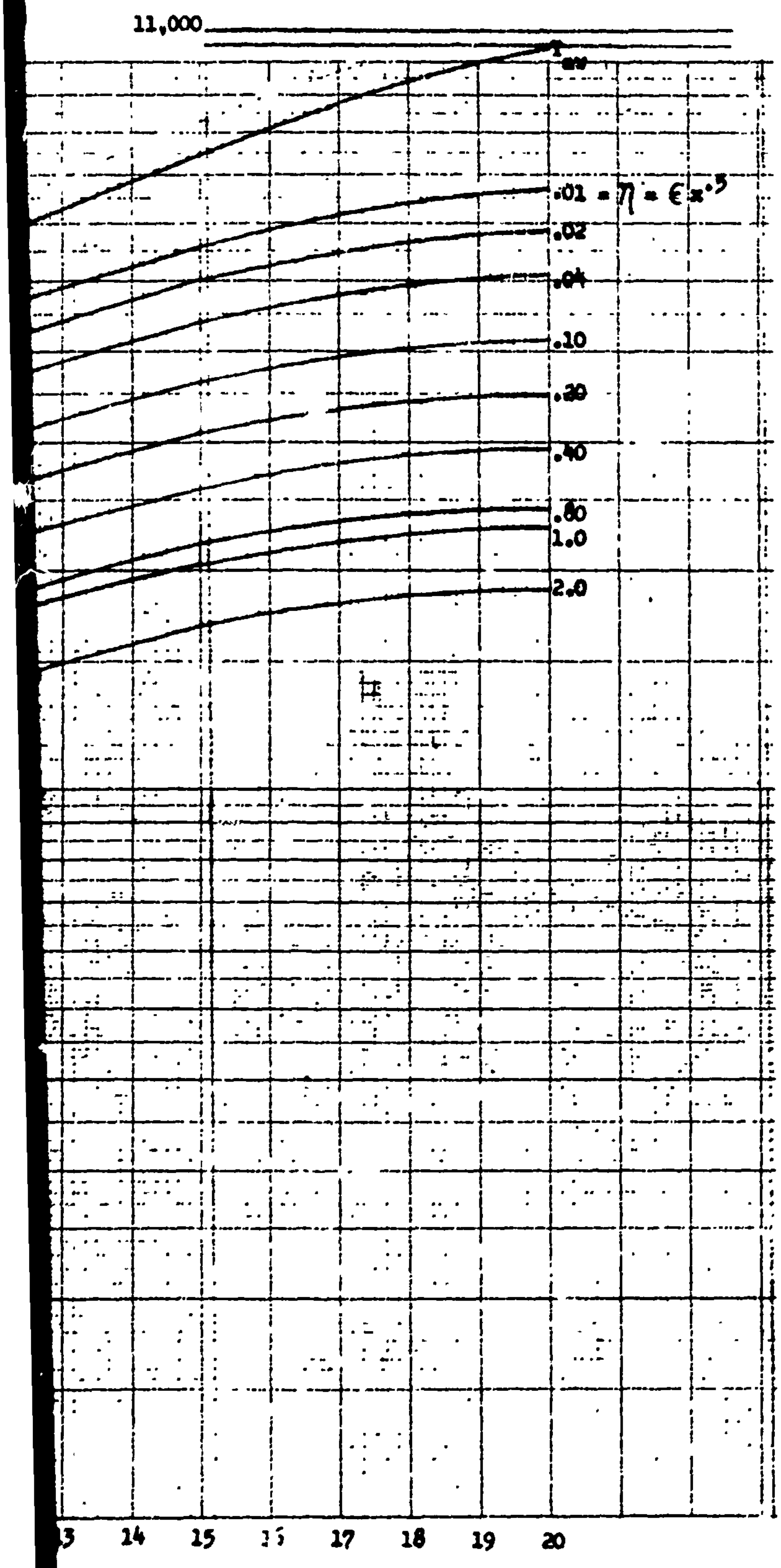
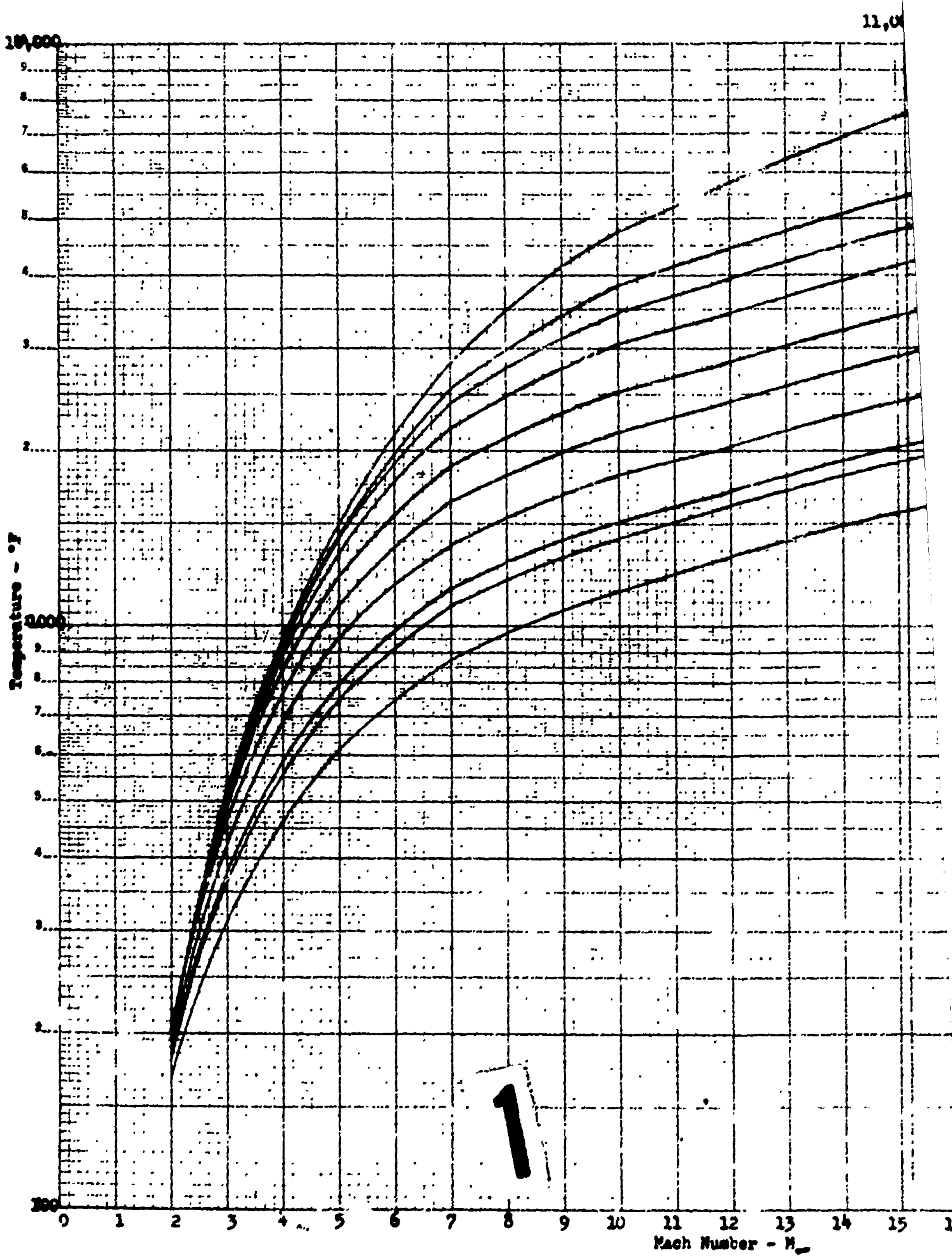


Figure 10 EQUILIBRIUM AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - LAMINAR BOUNDARY LAYER ALTITUDE = 60,000 FEET

3


11,0
10,000
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Temperature - °F
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Mach Number - M_∞
1

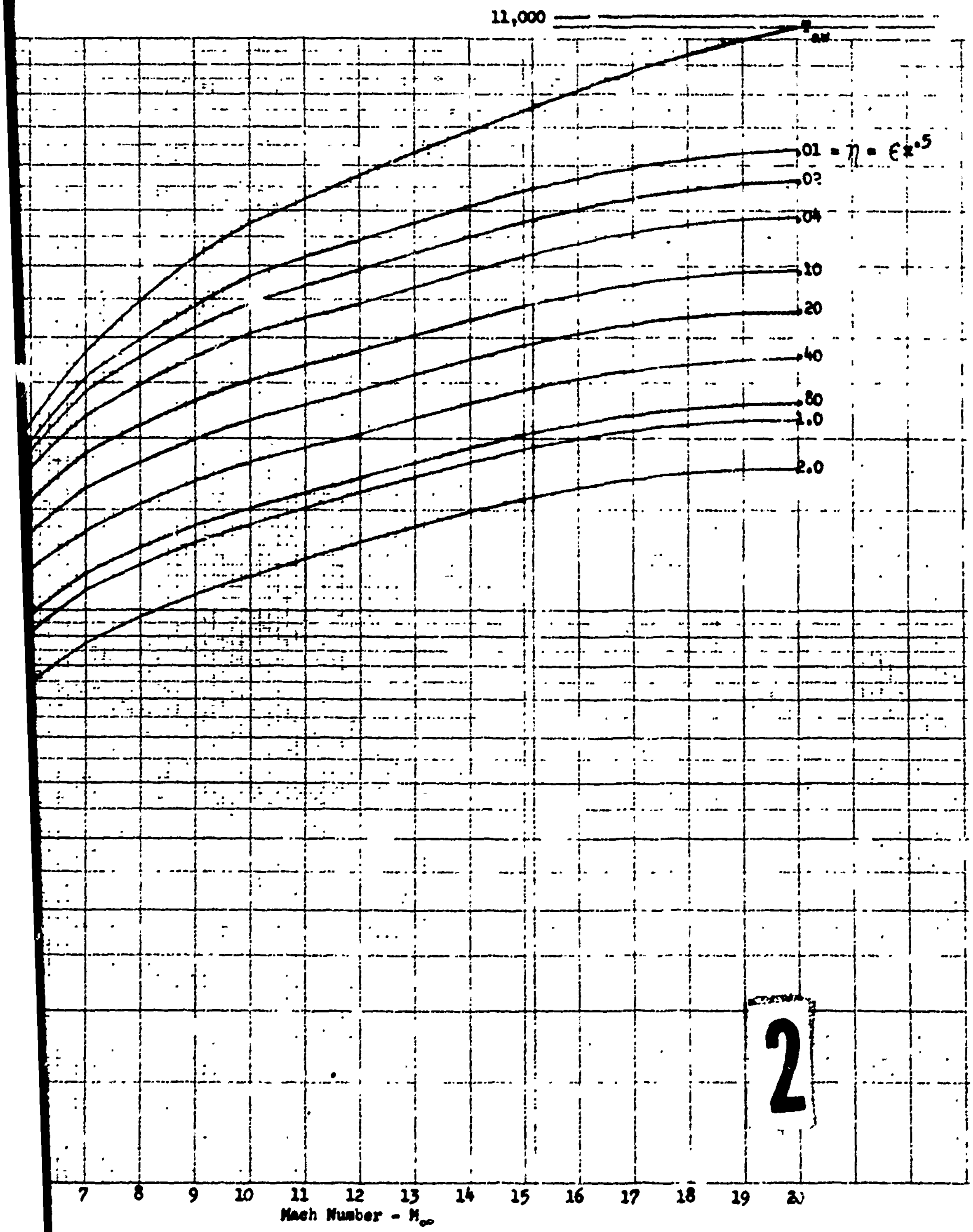


Figure 11 EQUILIBRIUM AN
TEMPERATURE VE
STANDARD DAY - LA
ALTITUDE =

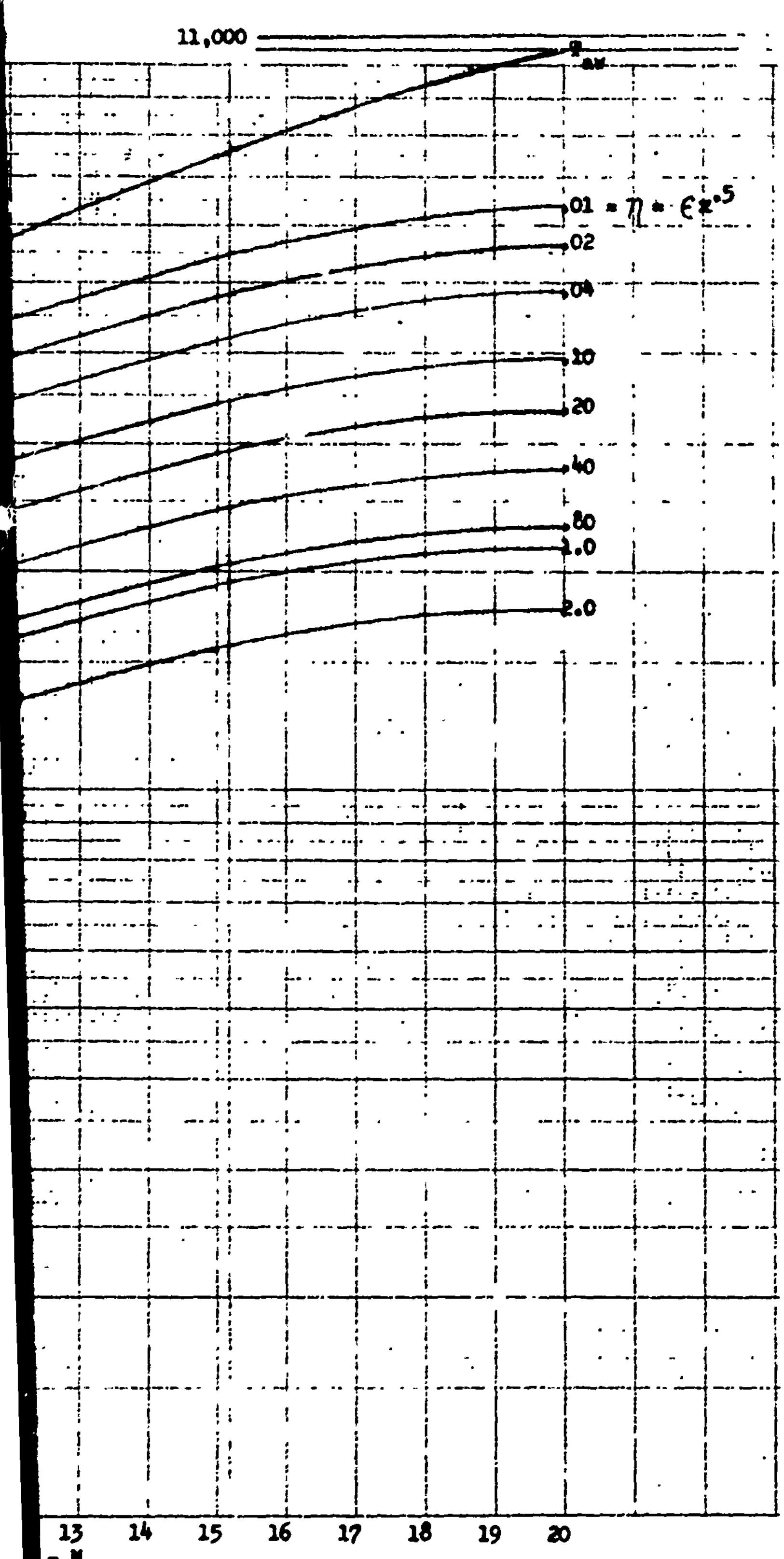
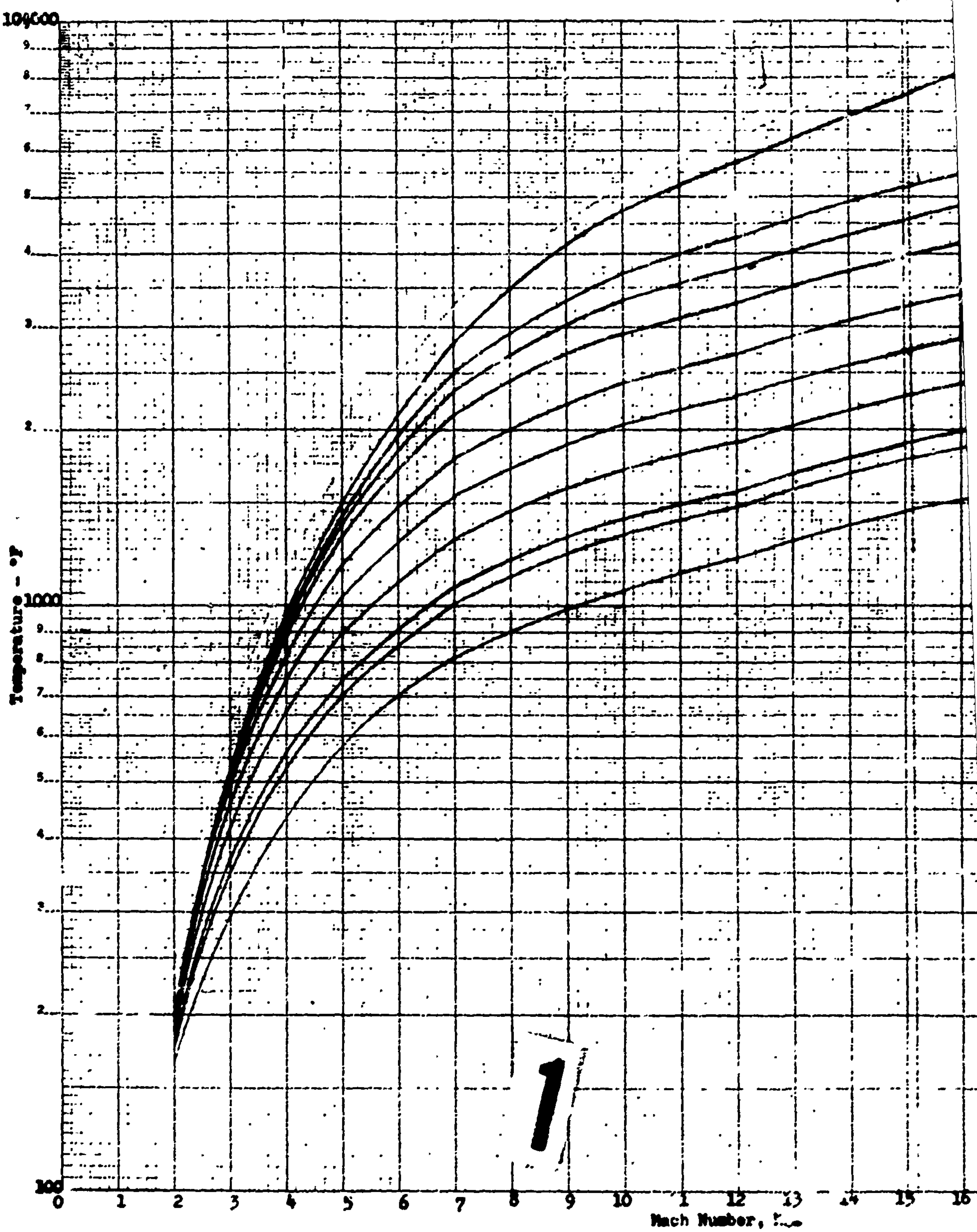


Figure 11 EQUILIBRIUM AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - LAMINAR BOUNDARY LAYER ALTITUDE = 70,000 FEET

3

11,000


Temperature - °F
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Mach Number,

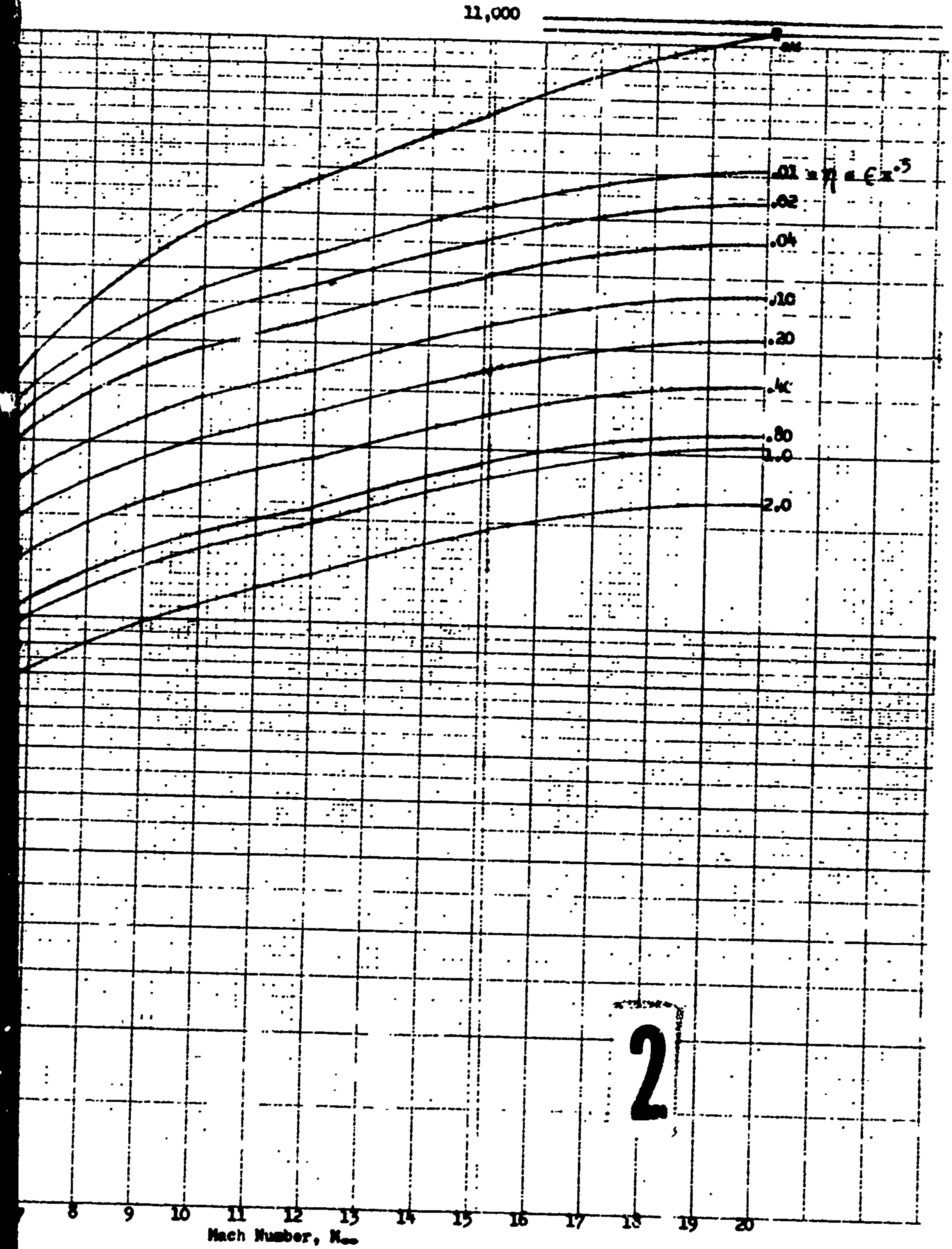


Figure 12 EQUILIBRIUM AND A
TEMPERATURE VERSU
STANDARD DAY - LAMINA
ALTITUDE = 80,

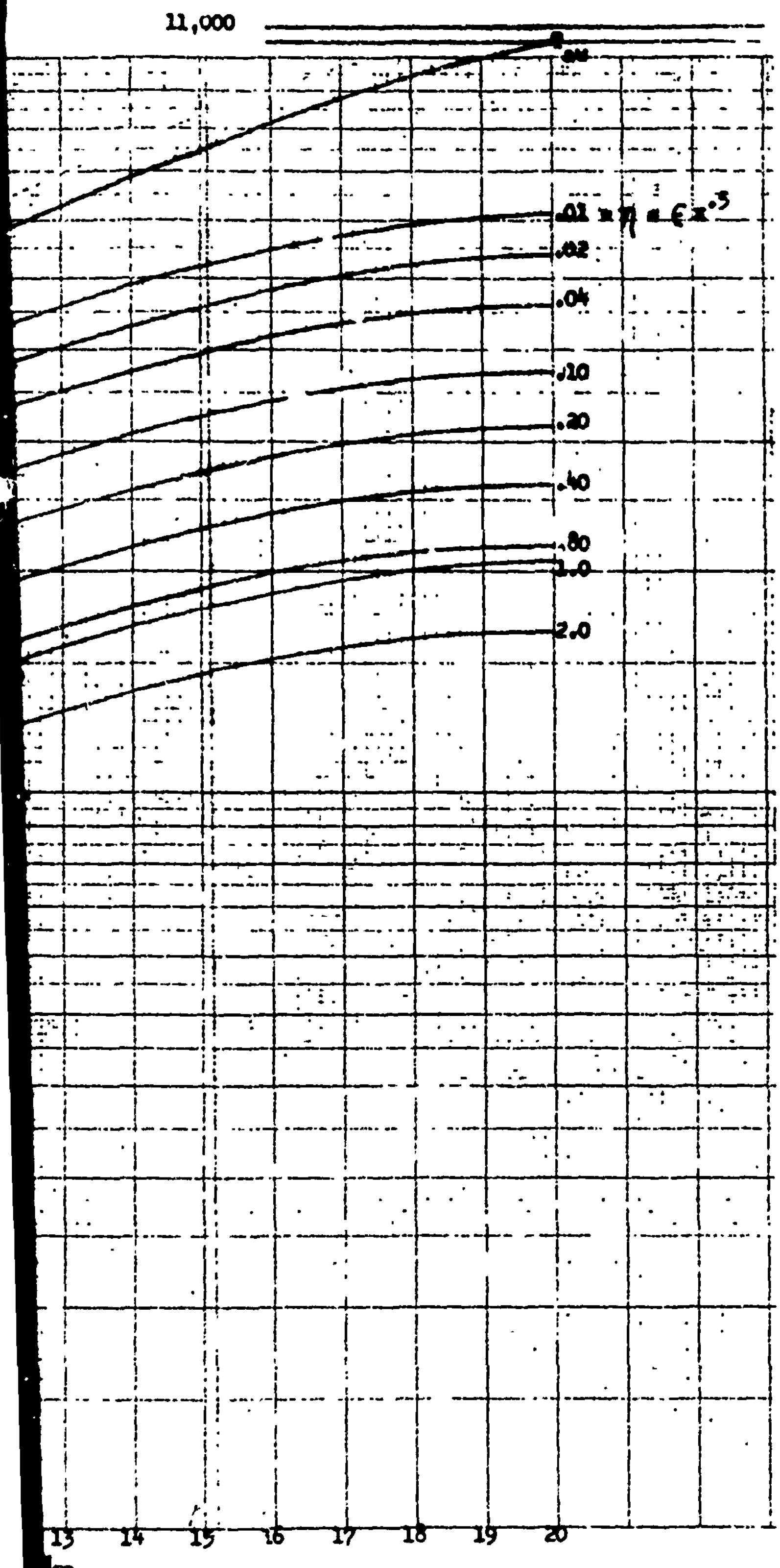
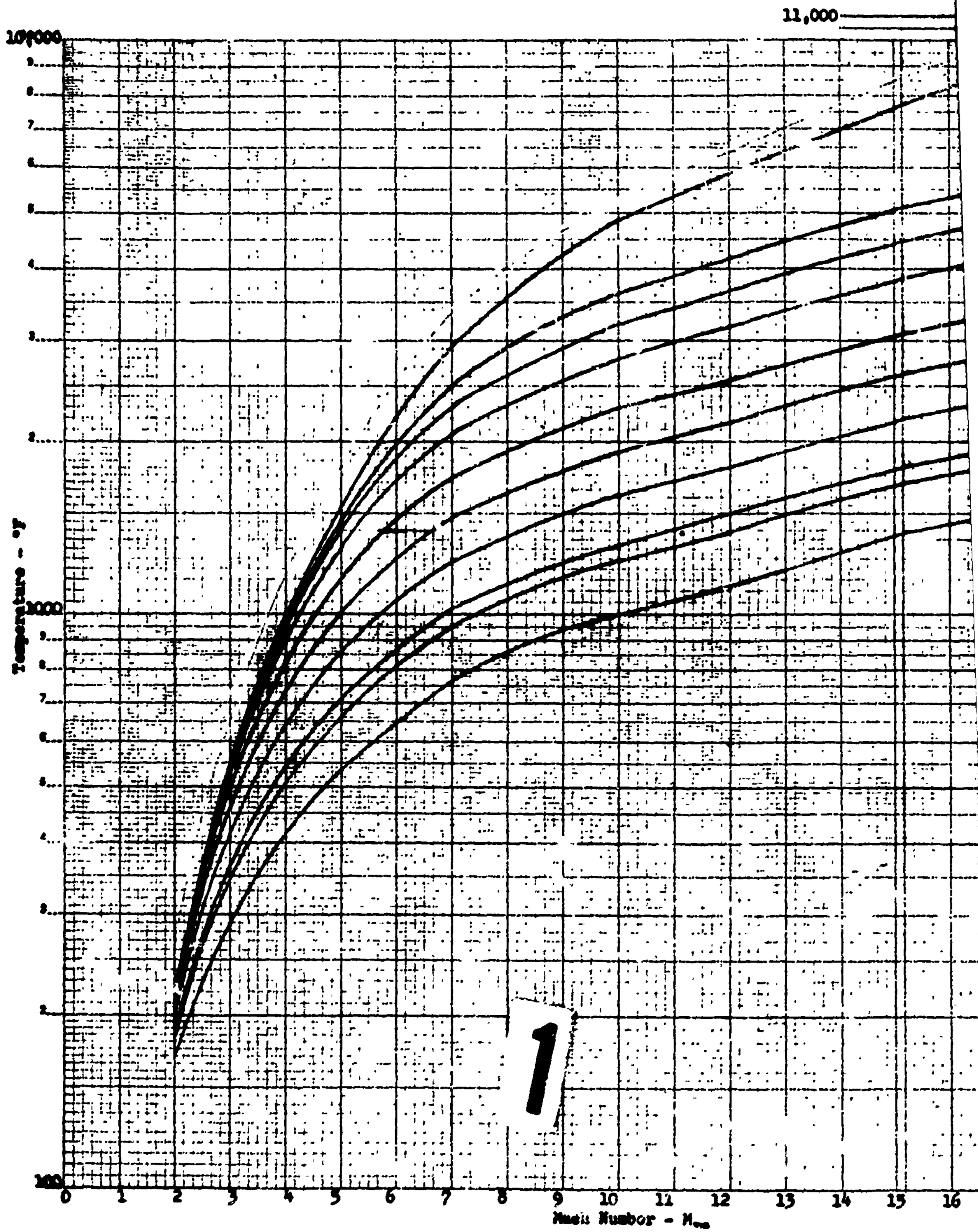


Figure 12 EQUILIBRIUM AND ADIABATIC WALL
TEMPERATURE VERSUS MACH NUMBER
STANDARD DAY - LAMINAR BOUNDARY LAYER
ALTITUDE = 80,000 FEET


11,000
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Temperature - °F
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15
16
Mach Number - M_∞
1

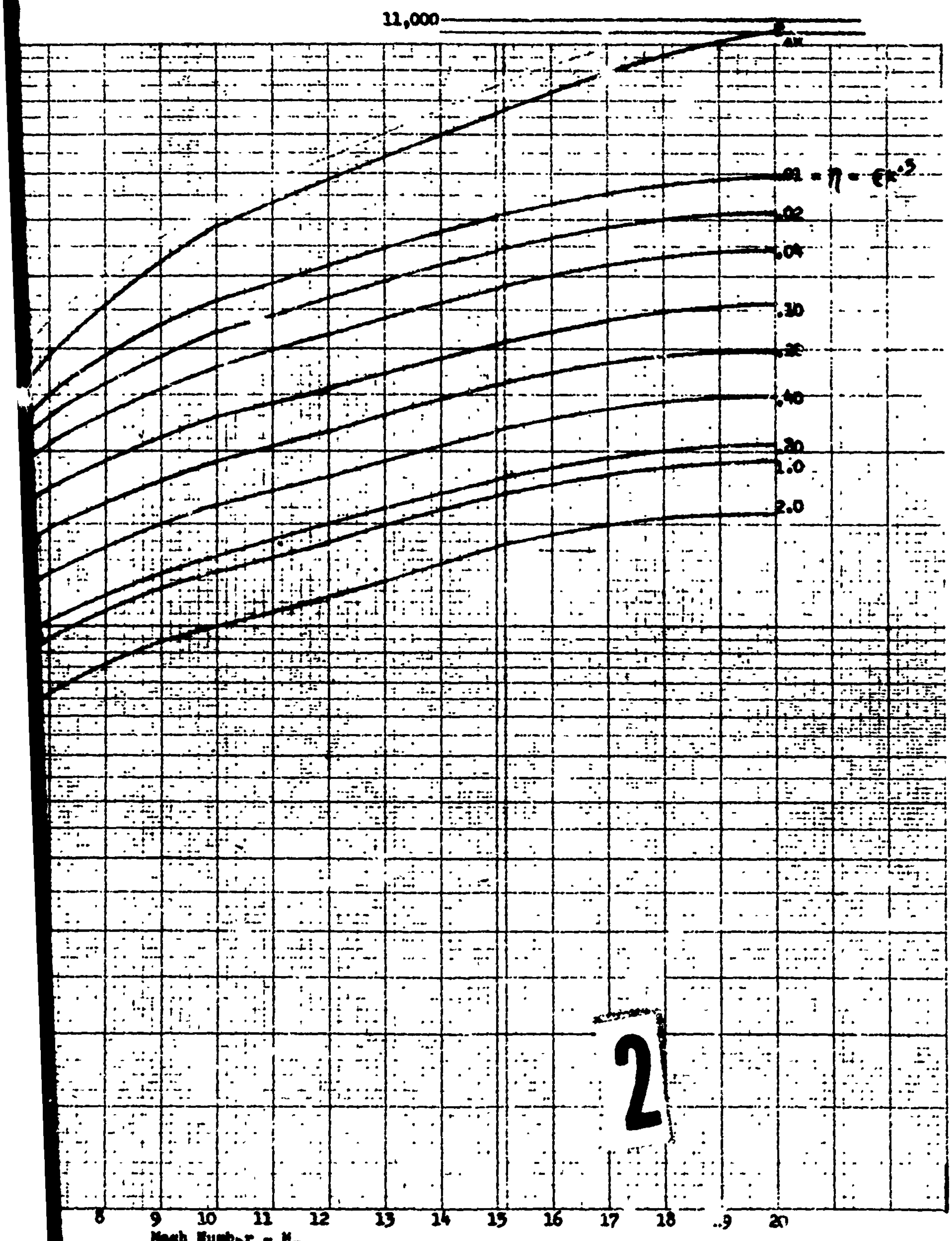


Figure 13 EQUILIBRIUM AND A
TEMPERATURE VERSU
STANDARD DAY - LAMIN
ALTITUDE = 90

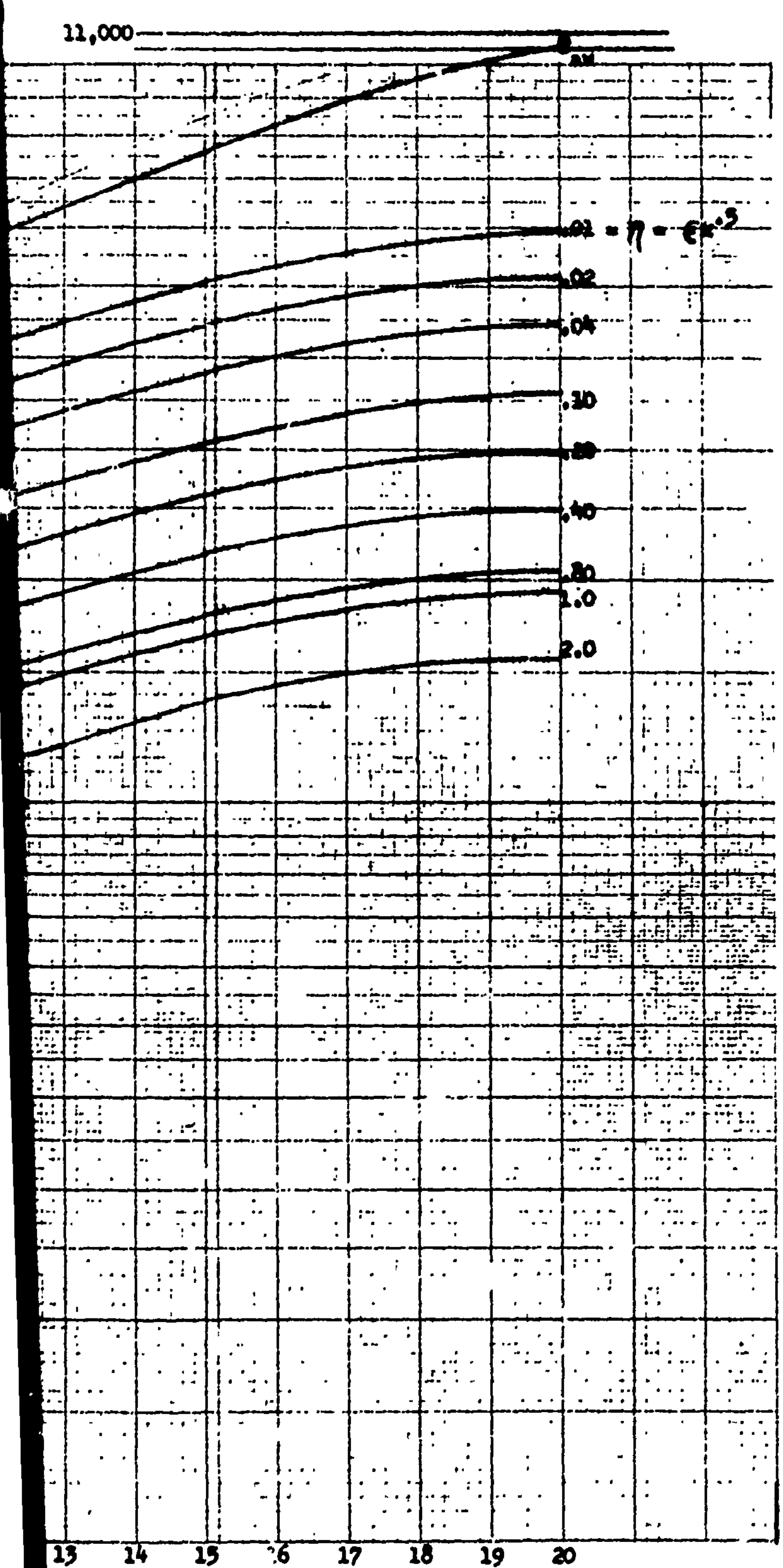
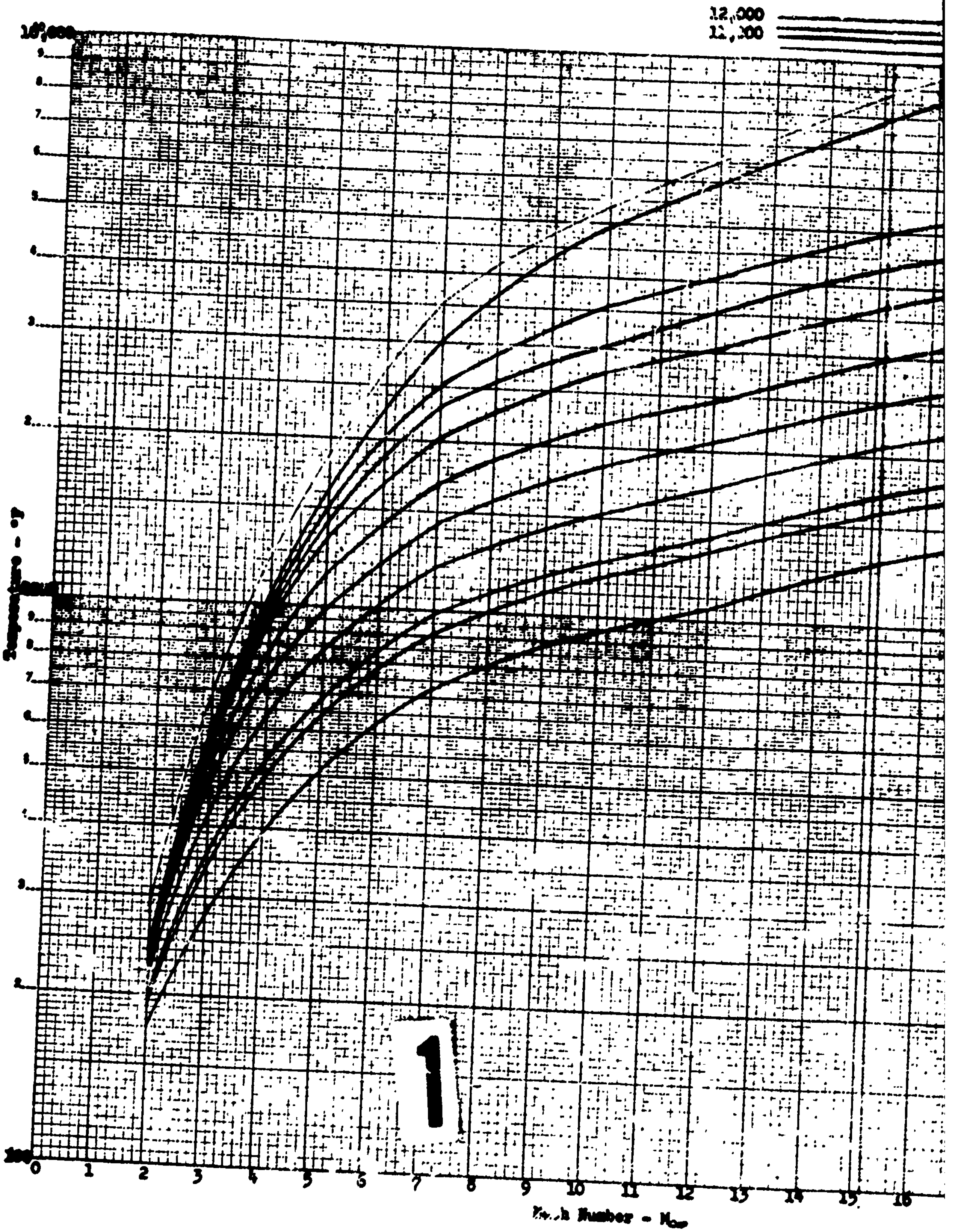


Figure 13 EQUILIBRIUM AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER
STANDARD DAY - LAMINAR BOUNDARY LAYER
ALTITUDE = 90,000 FEET

3


12,000
10,000
100
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Mach Number - M_∞

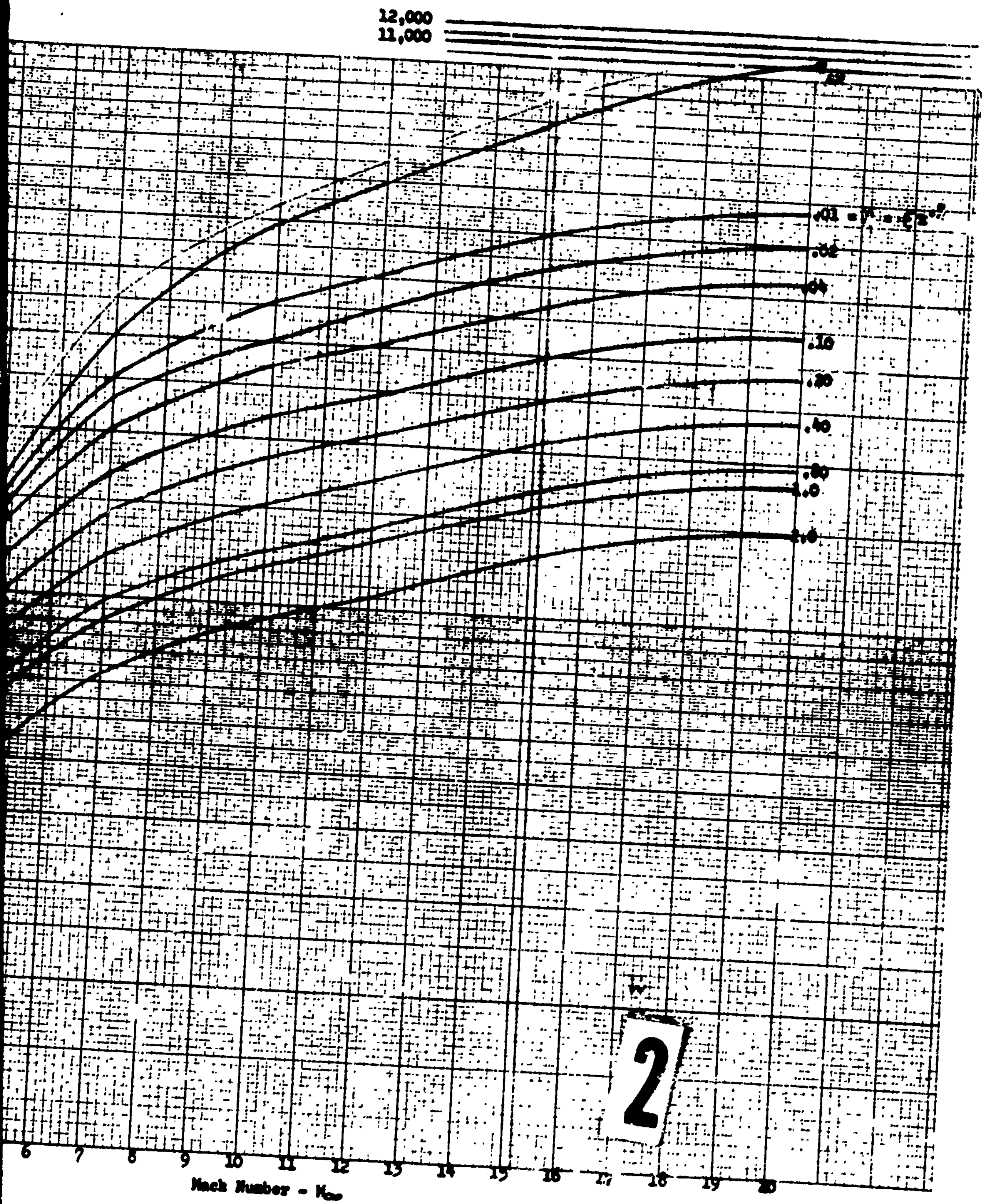


Figure 14 EQUILIBR
TEMPERAT
STANDARD DA
ALTIT

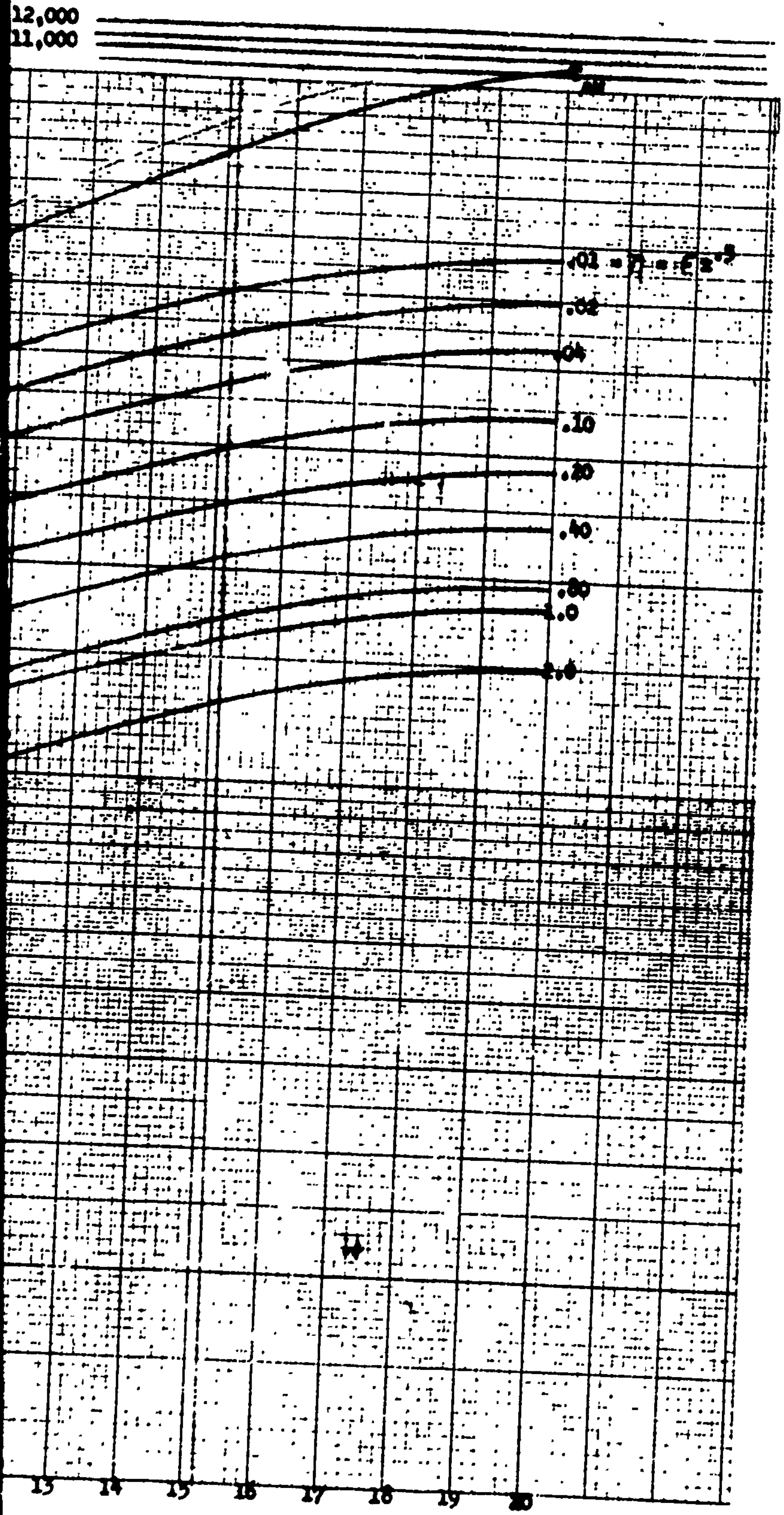
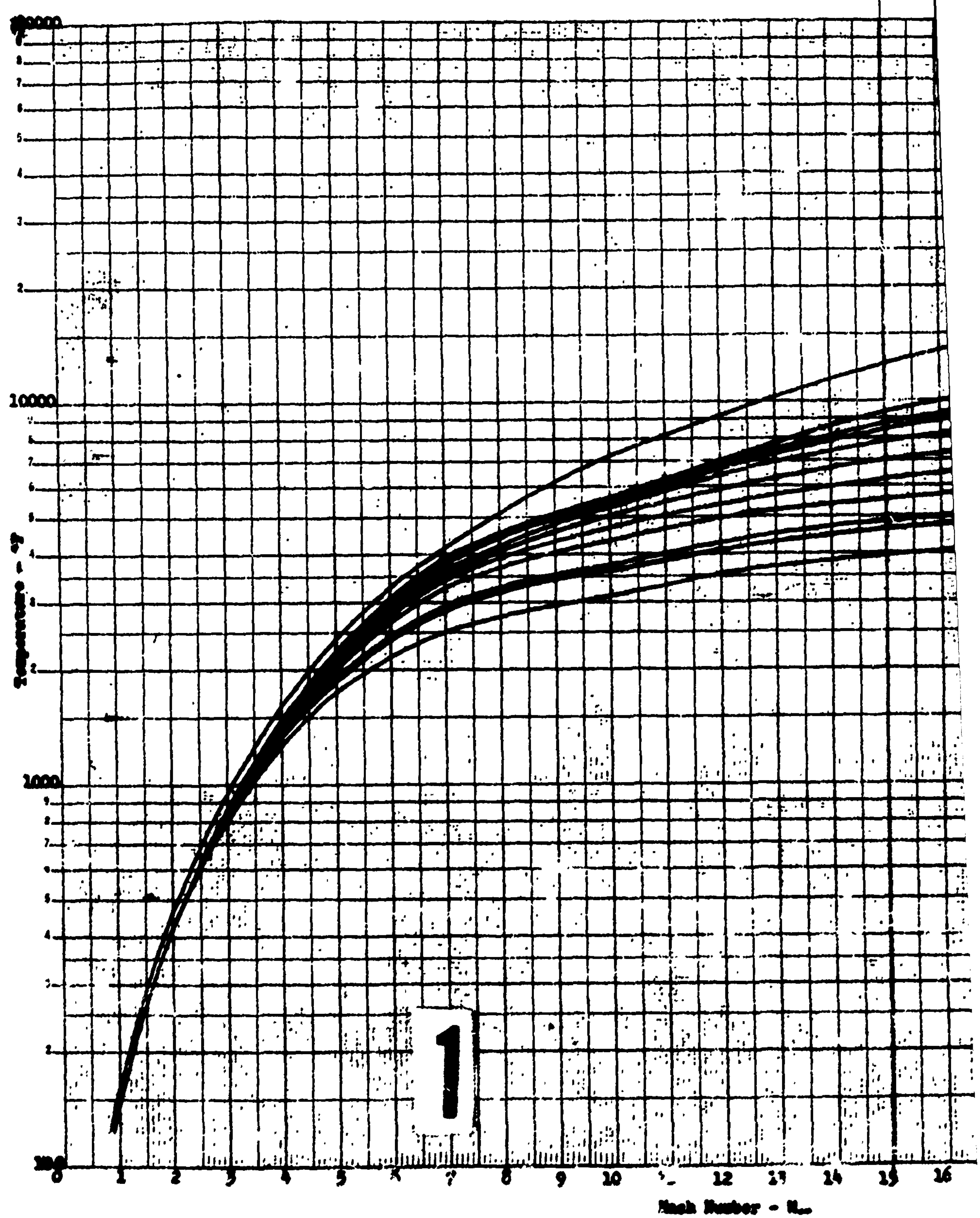


Figure 14 EQUILIBRIUM AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - LAMINAR BOUNDARY LAYER ALTITUDE = 100,000 FEET

3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14
16
Mach Number - M

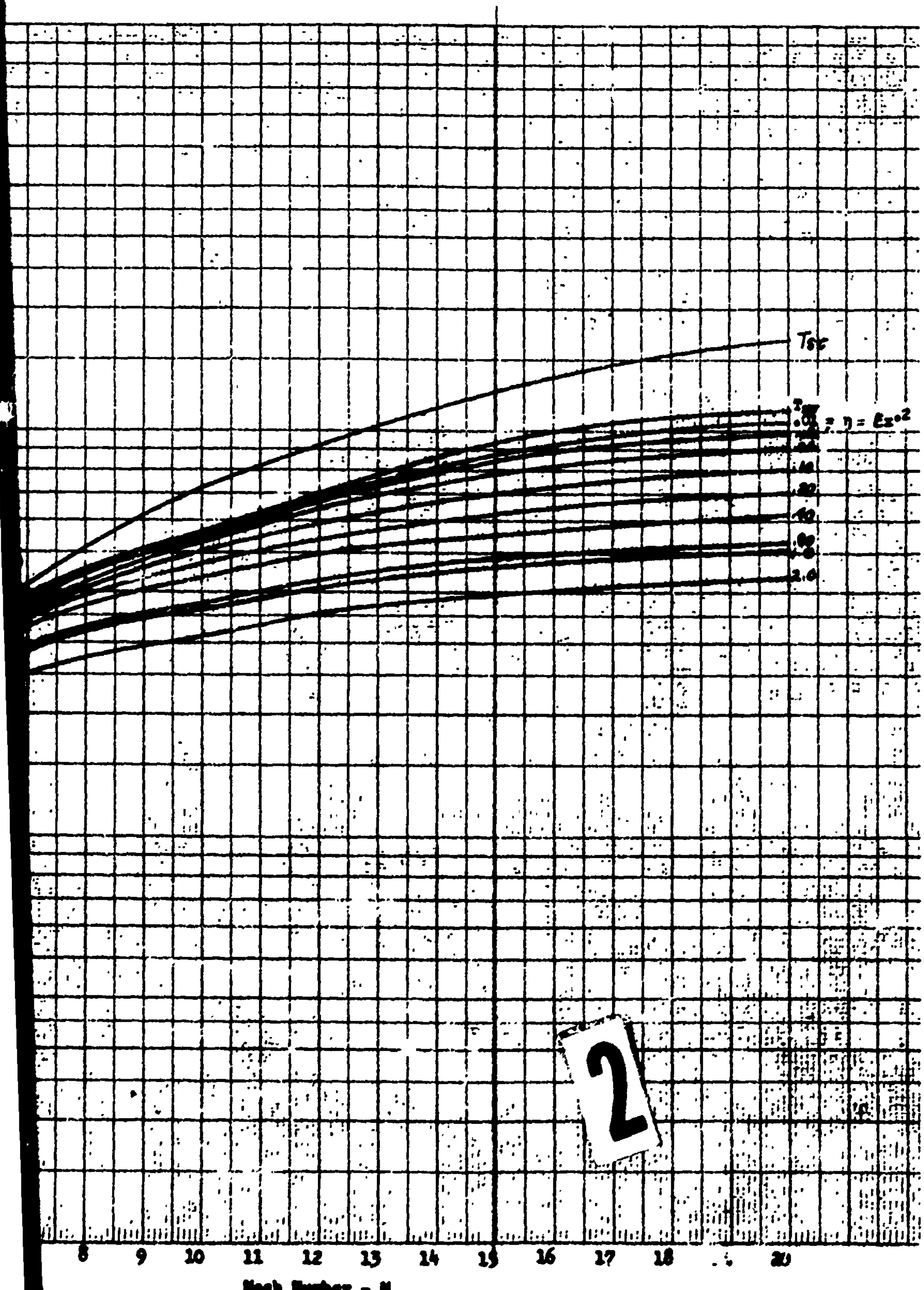


Figure 15 EQUILIBRIUM, STAGNATI
WALL TEMPERATURE VI
STANDARD DAY - TURBU
ALTITUDE = 0

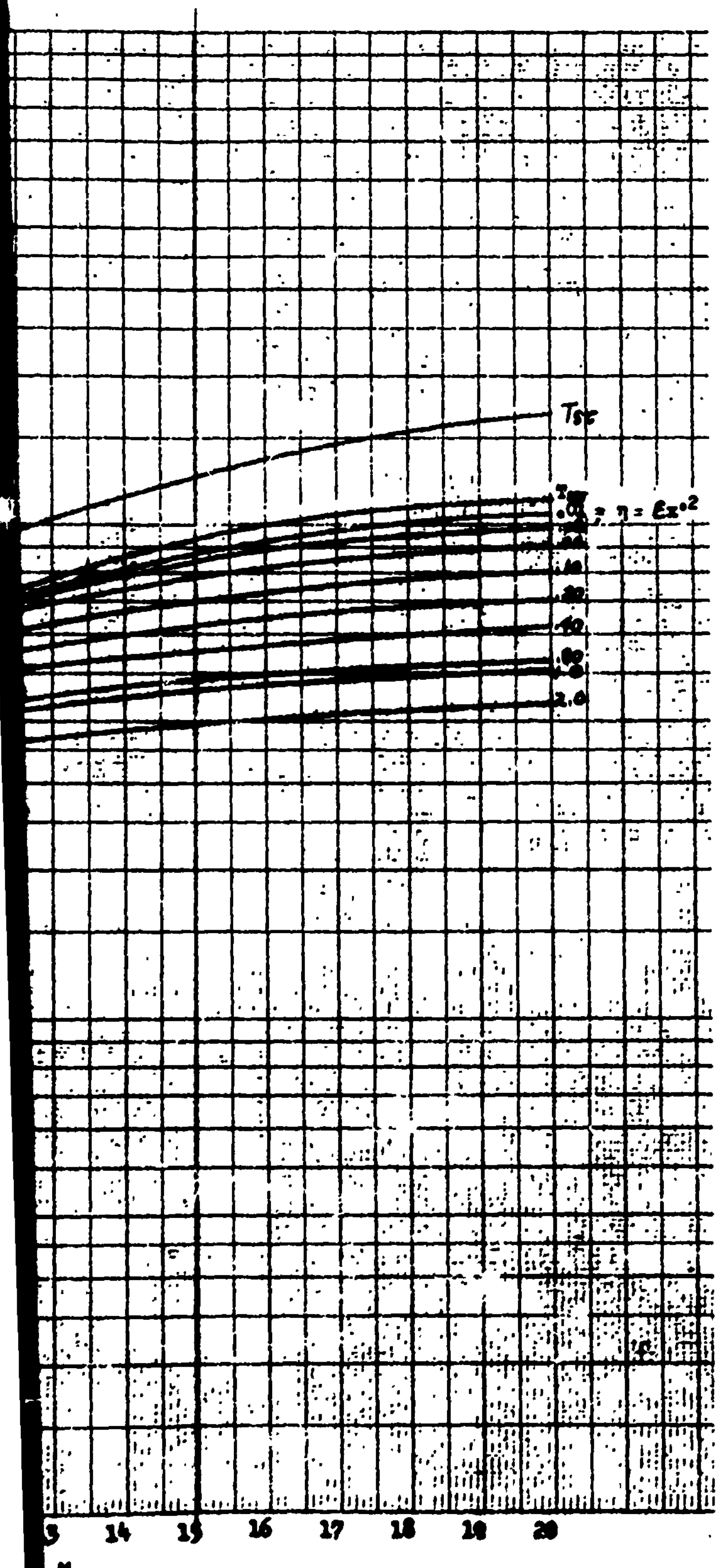
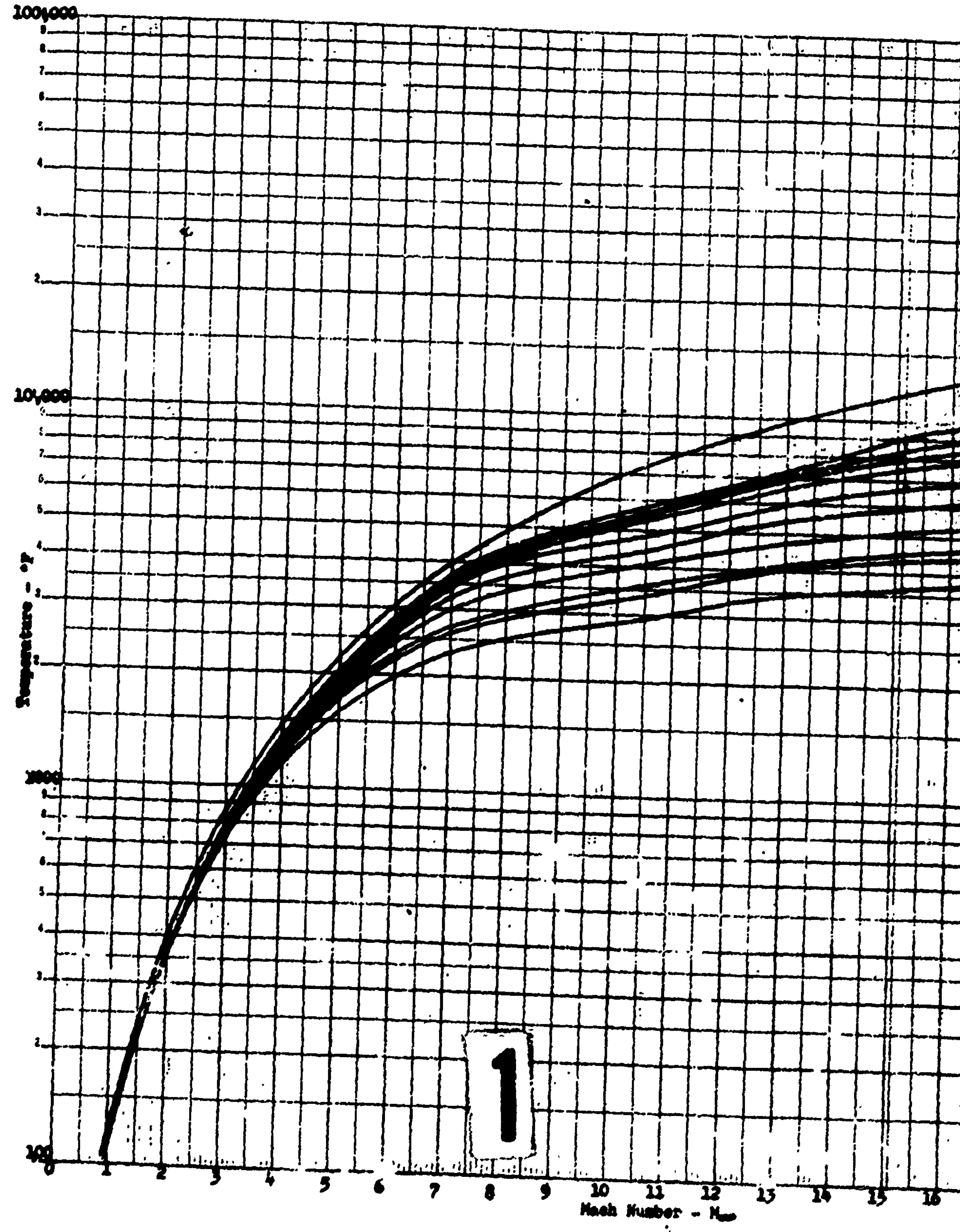


Figure 15 EQUILIBRIUM, STAGNATION, AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - TURBULENT BOUNDARY LAYER ALTITUDE = 0 FEET

3


100,000
10,000
1000
100
Temperature - °F
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15
16
Mach Number - M_∞
1

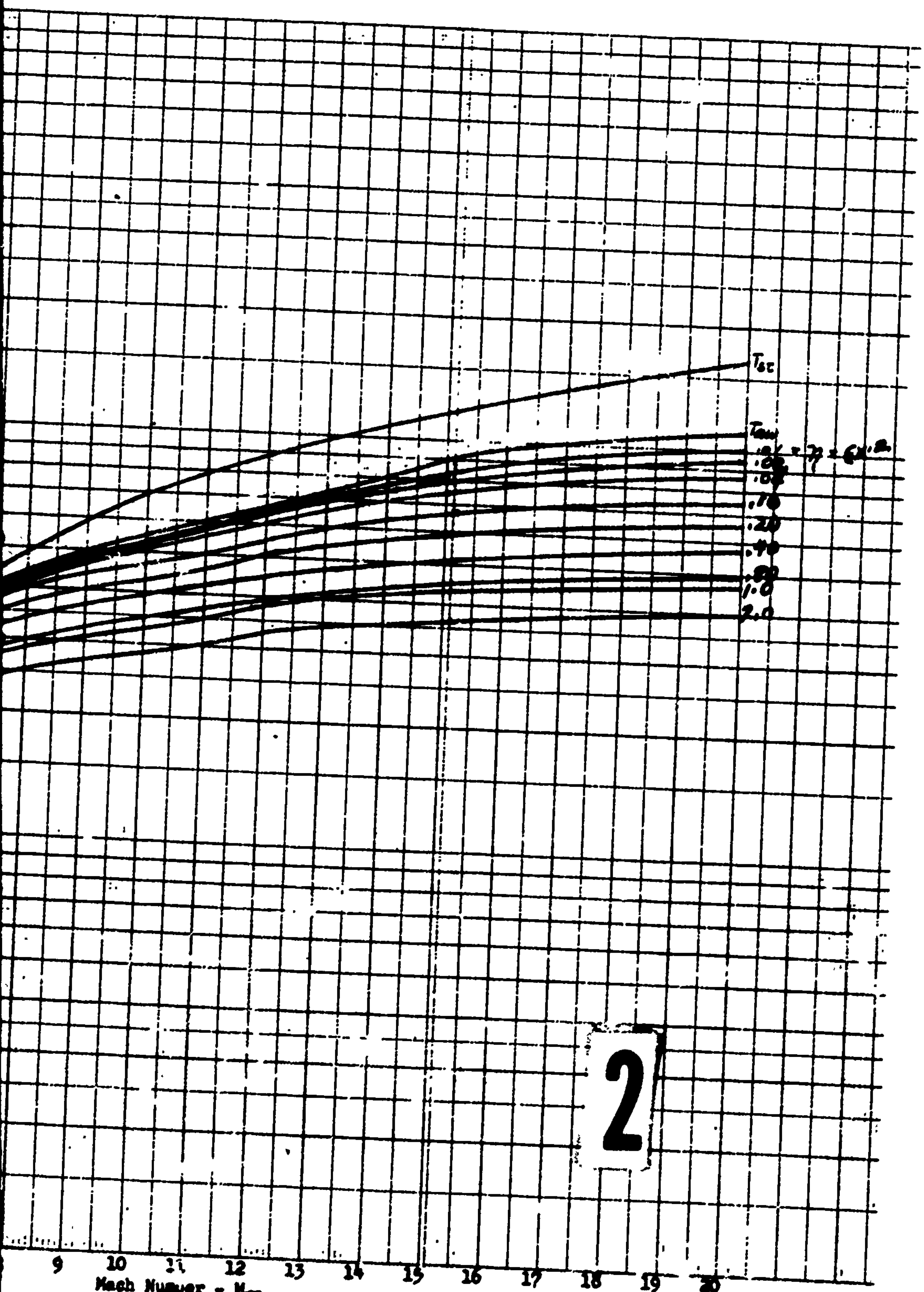


Figure 16 EQUILIBRIUM, STAGNATIO
WALL TEMPERATURE VER
STANDARD DAY - TURBULE
ALTITUDE = 10,000

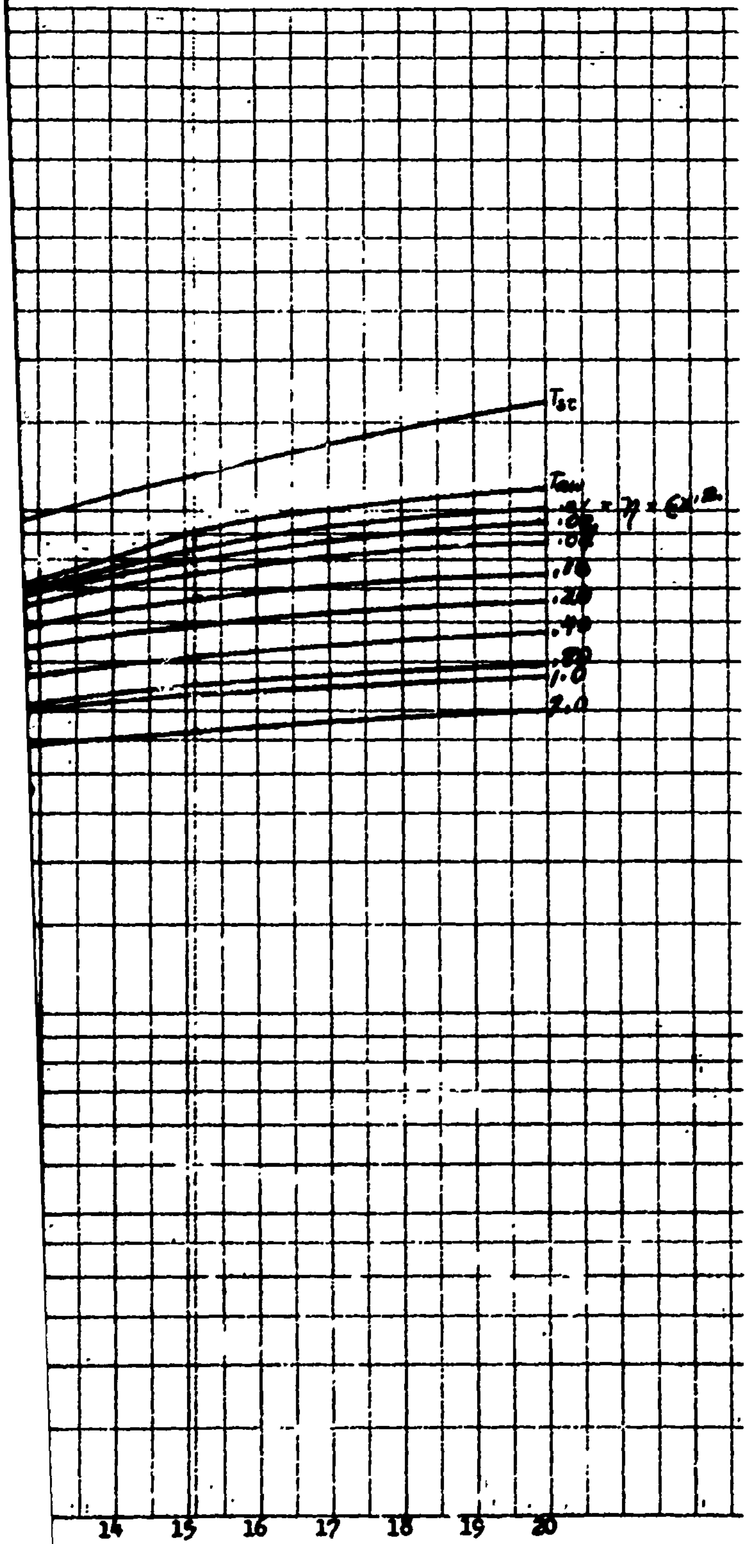
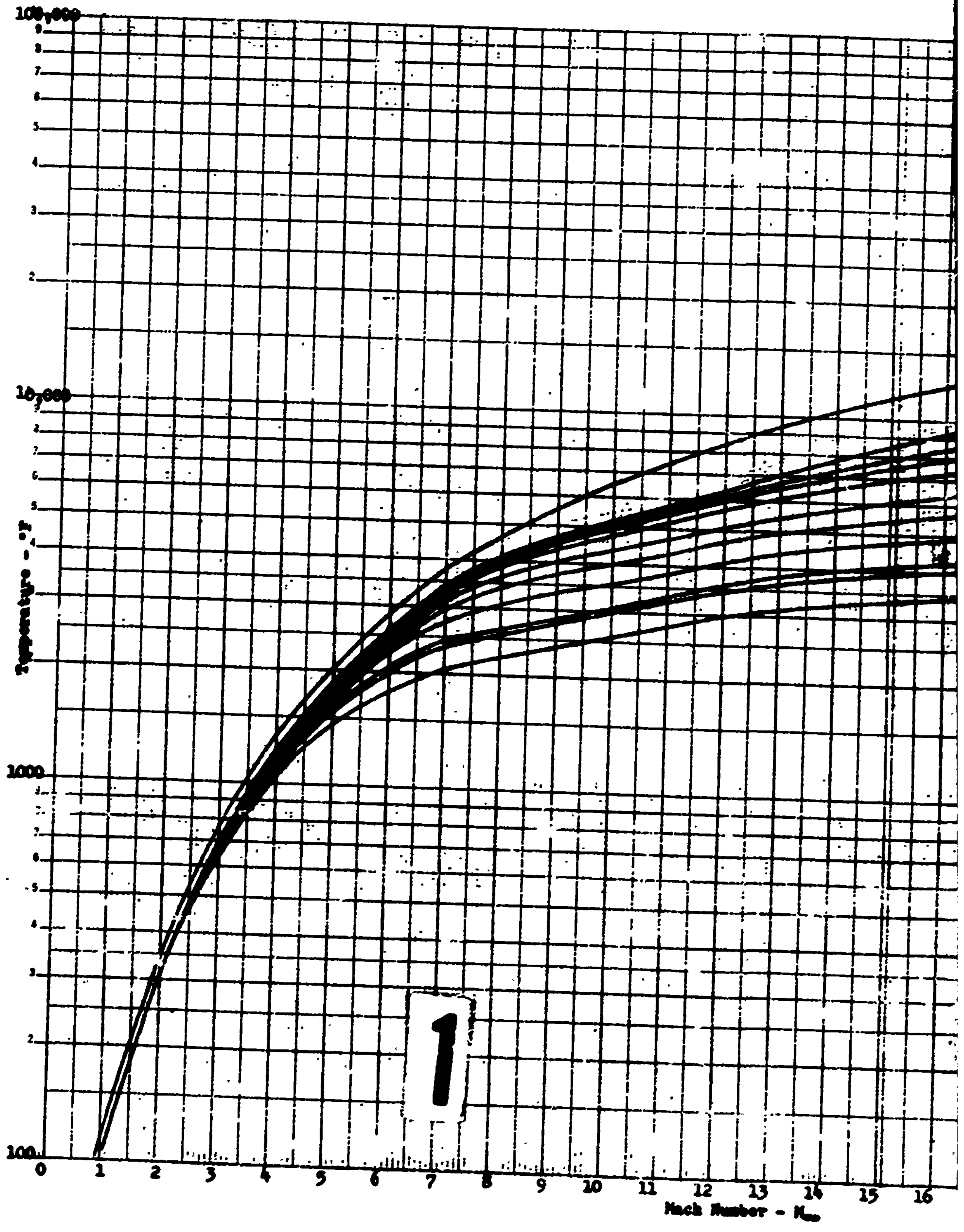


Figure 16 EQUILIBRIUM, STAGNATION, AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - TURBULENT BOUNDARY LAYER ALTITUDE = 10,000 FEET

3


100,000
10,000
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100
Temperature °F
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16
Mach Number - M_∞

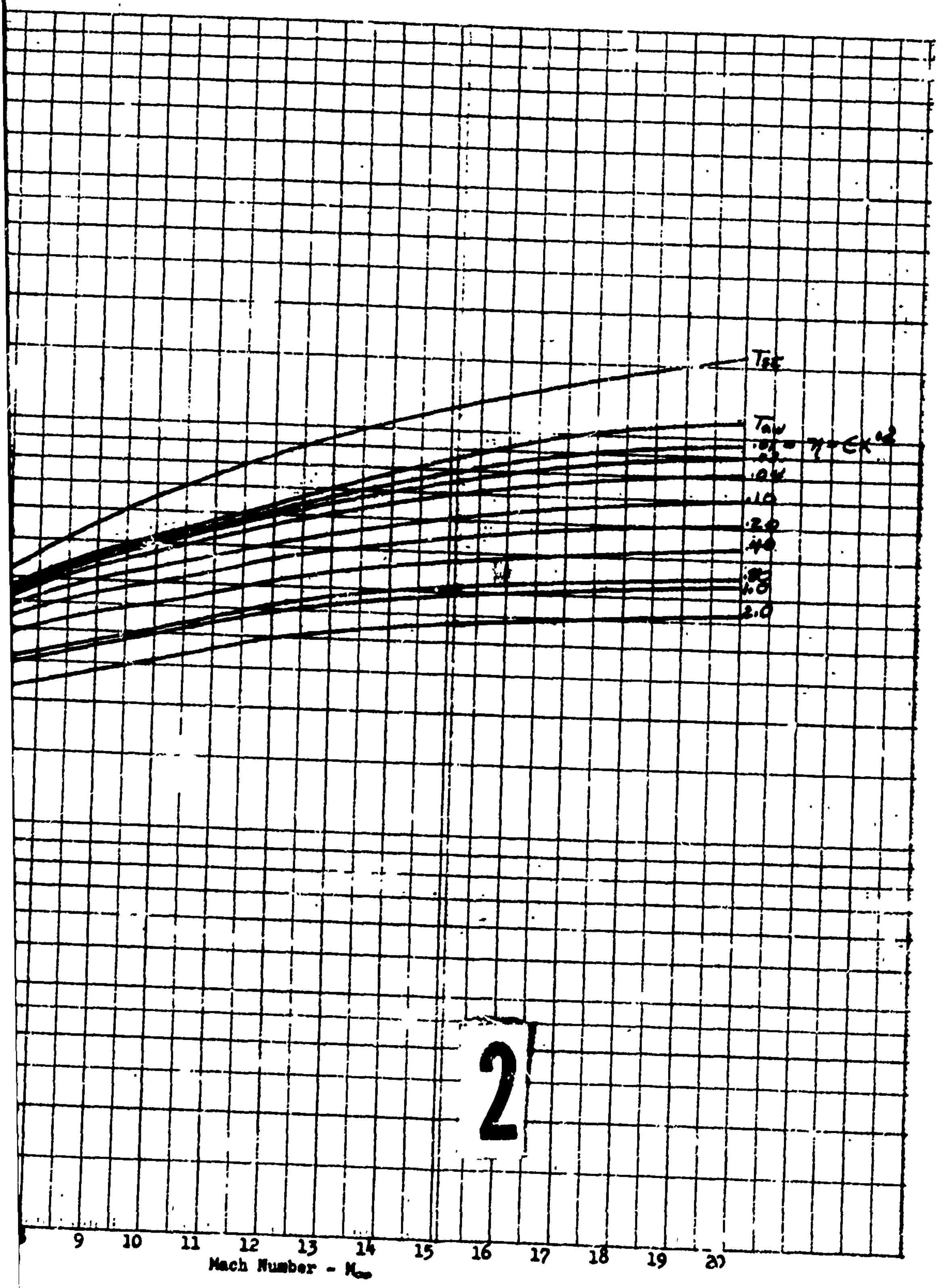


Figure 17 EQUILIBRIUM, STAGNAT
WALL TEMPERATURE V
STANDARD DAY - TURBU
ALTITUDE = 20,0

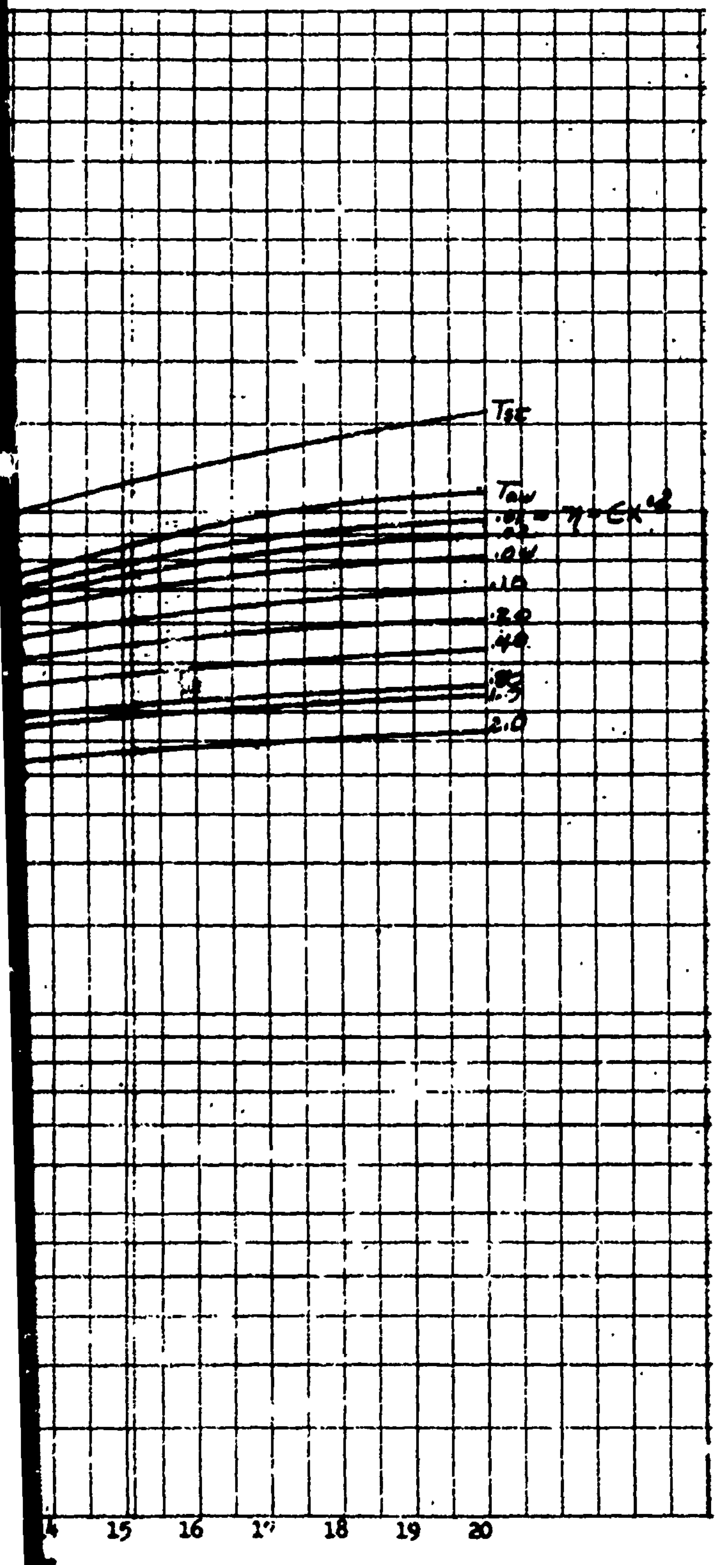
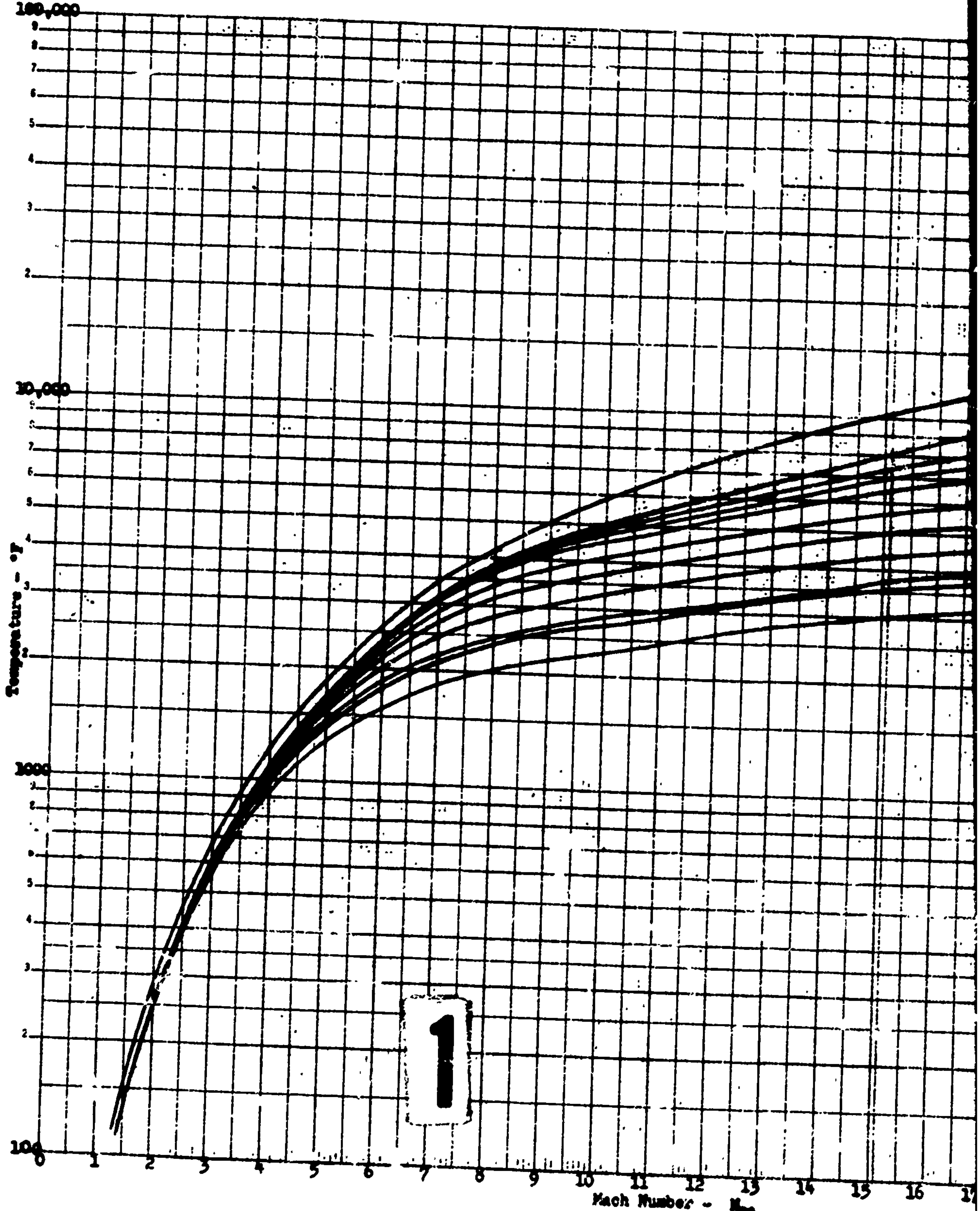


Figure 17 EQUILIBRIUM, STAGNATION, AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - TURBULENT BOUNDARY LAYER ALTITUDE = 20,000 FEET

3


100,000
10,000
1000
100
Temperature - °F
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16
17
Mach Number - M_∞
1

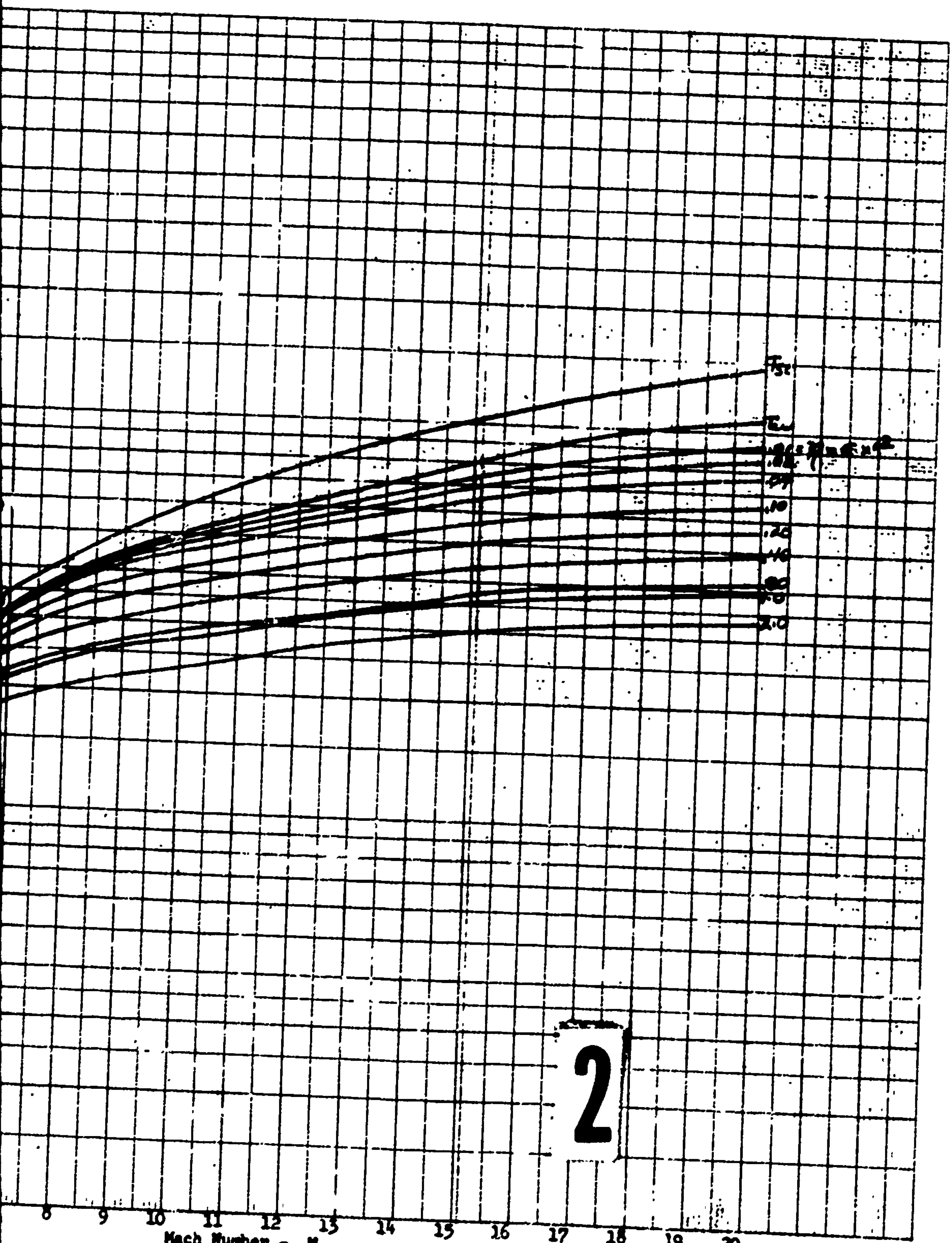


Figure 18 EQUILIBRIUM, STA
WALL TEMPERATU
STANDARD DAY - T
ALTITUDE =

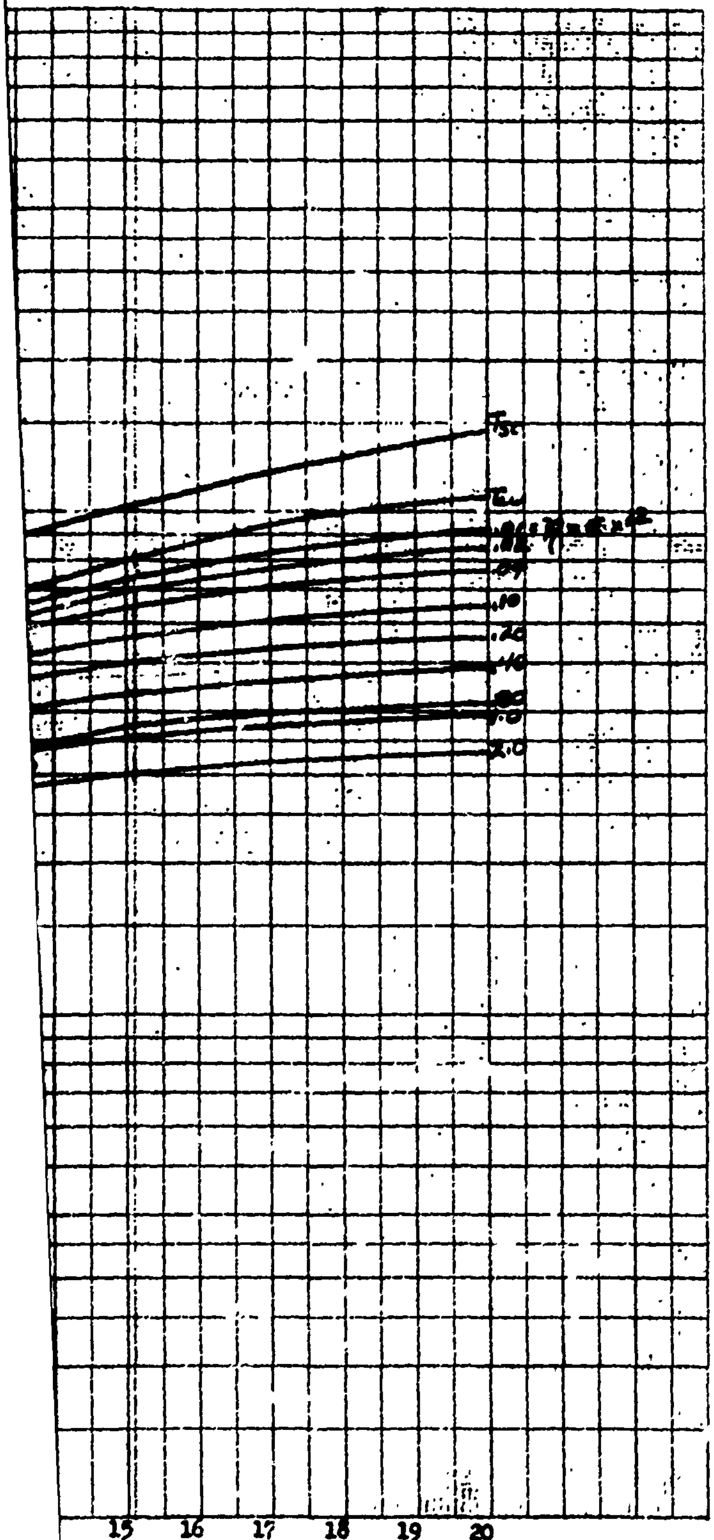
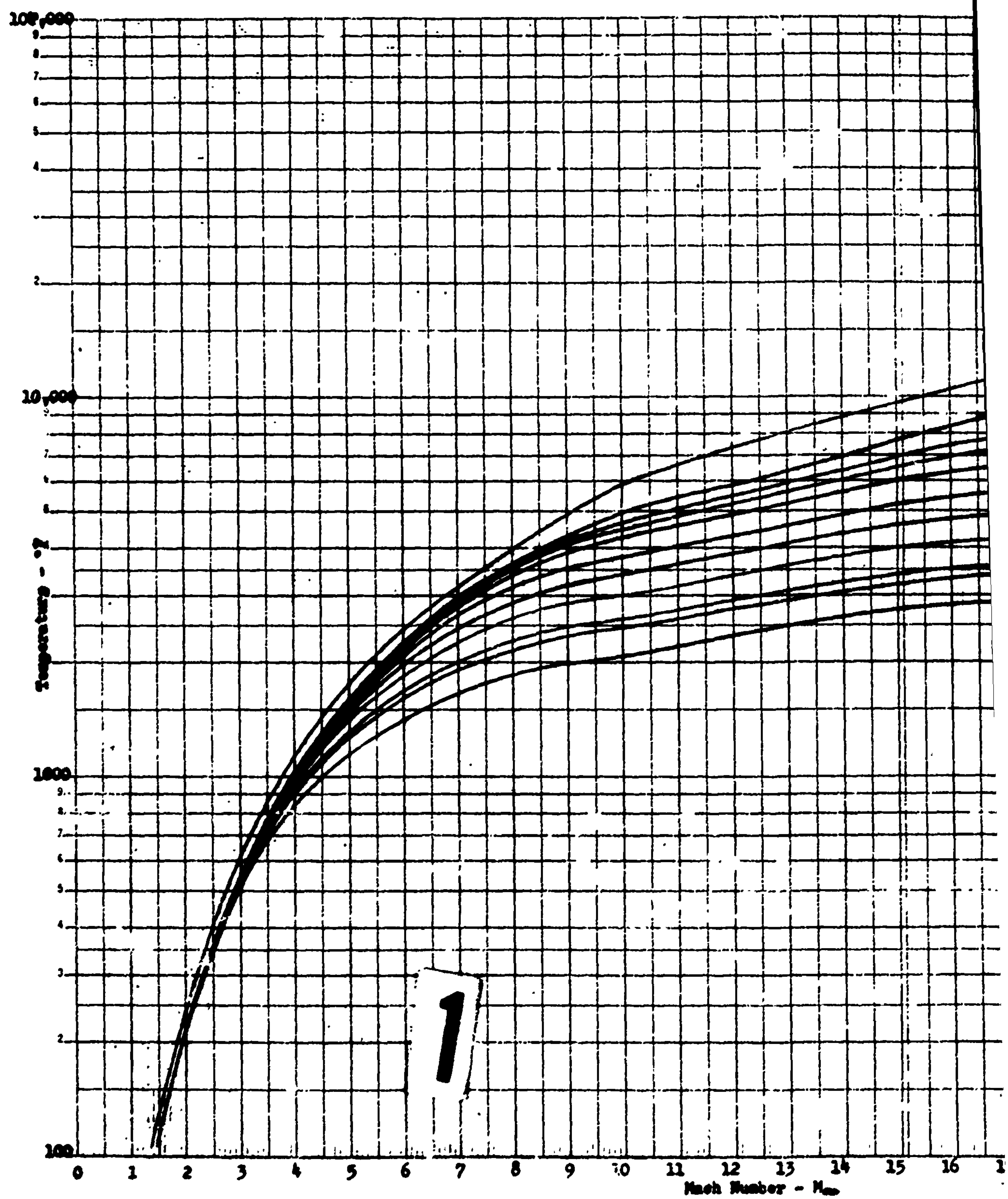


Figure 18 EQUILIBRIUM, STAGNATION, AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - TURBULENT BOUNDARY LAYER ALTITUDE = 30,000 FEET


100,000
10,000
1000
100
Temperature - °F
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11
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15
16
Mach Number - M∞
1

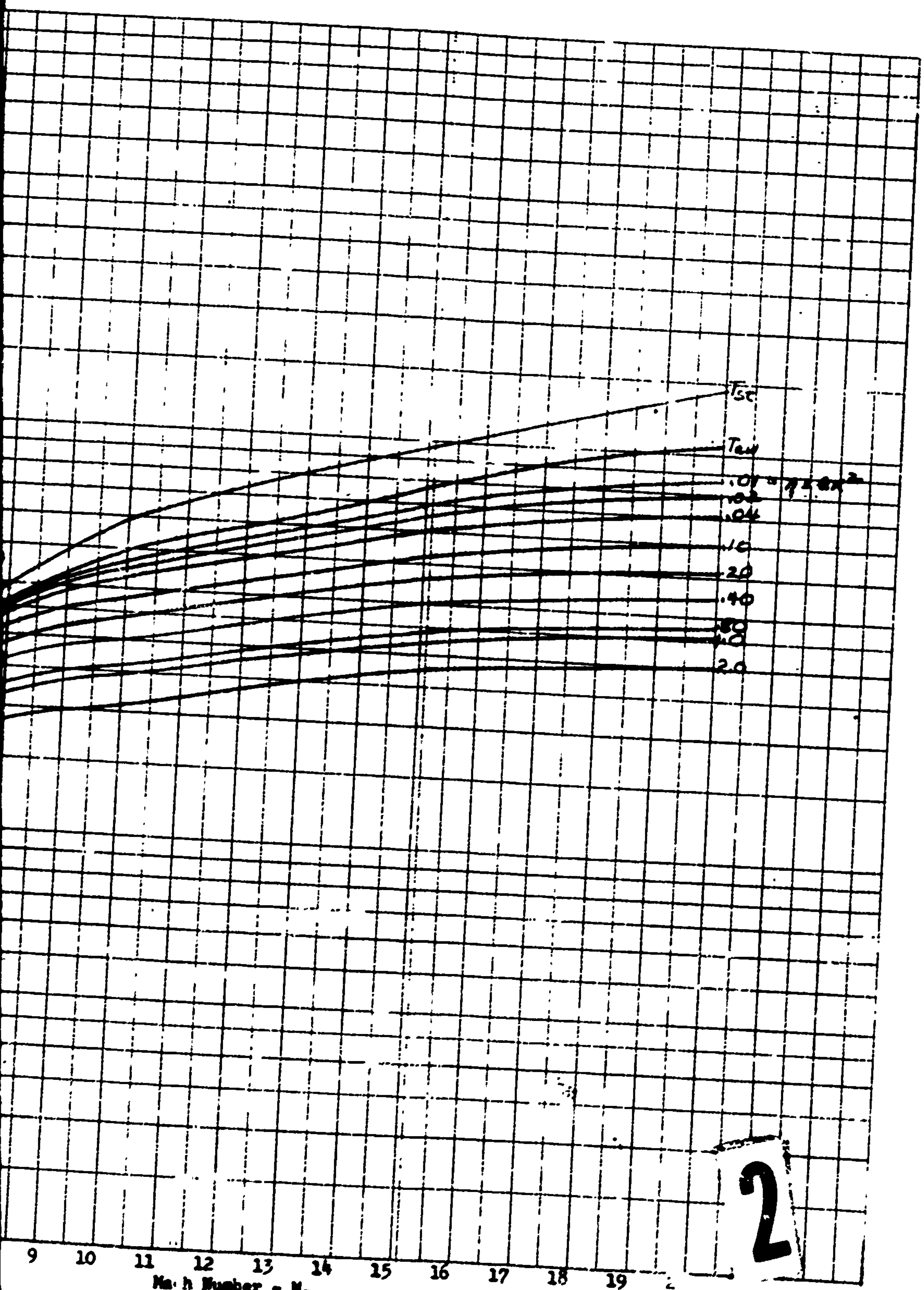


Figure 19 EQUIL RIUM, STAGNATI
WALL TEMPERATURE VE
STANDARD DAY - TURBUL
ALTITUDE = 40,00

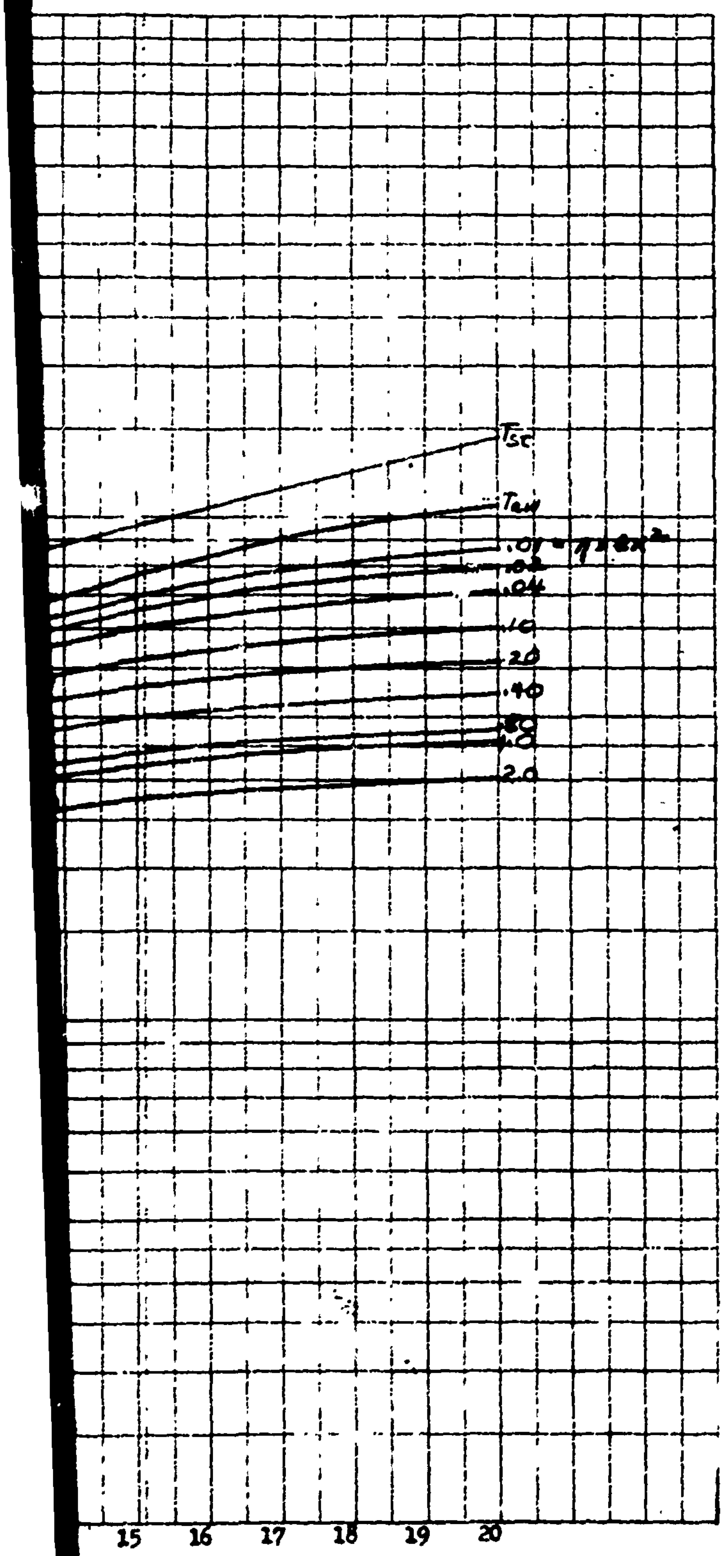
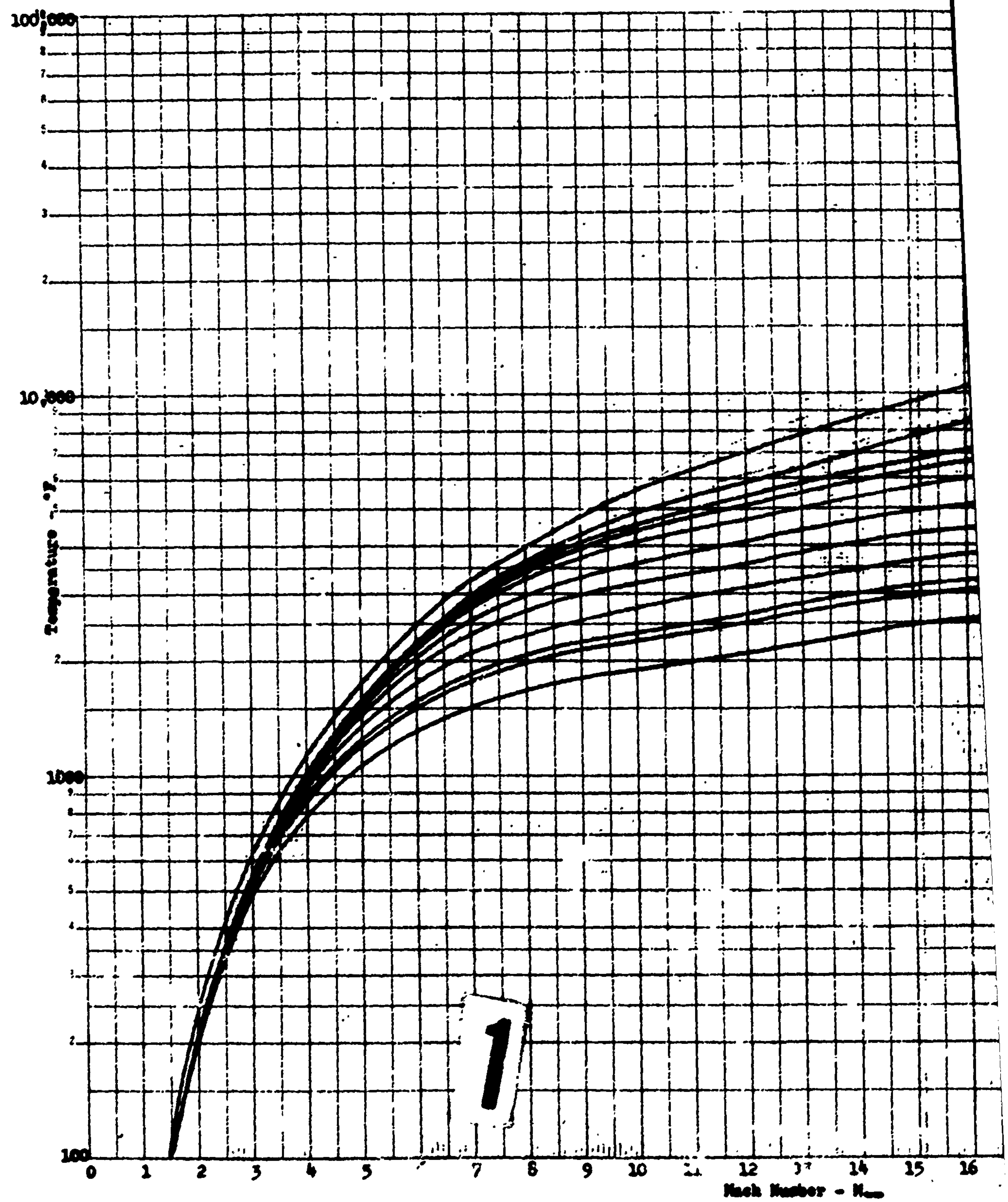


Figure 19 EQUILIBRIUM, STAGNATION, AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - TURBULENT BOUNDARY LAYER ALTITUDE = 40,000 FEET

3


100,000
10,000
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100
Temperature - °F
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Mach Number - M
1

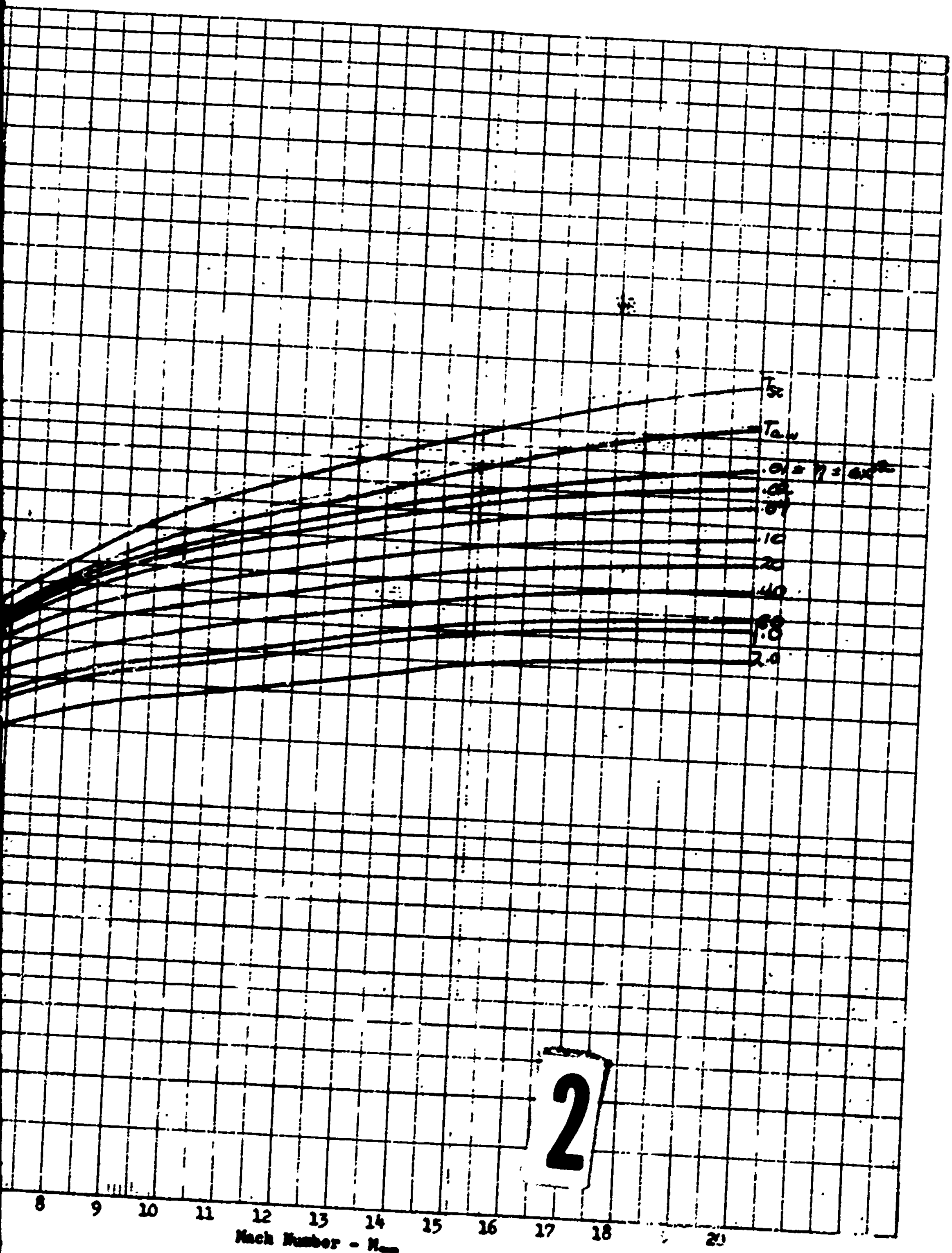


Figure 20 EQUILIBRIUM, STAG
WALL TEMPERATUR
STANDARD DAY - TU
ALTITUDE = 5

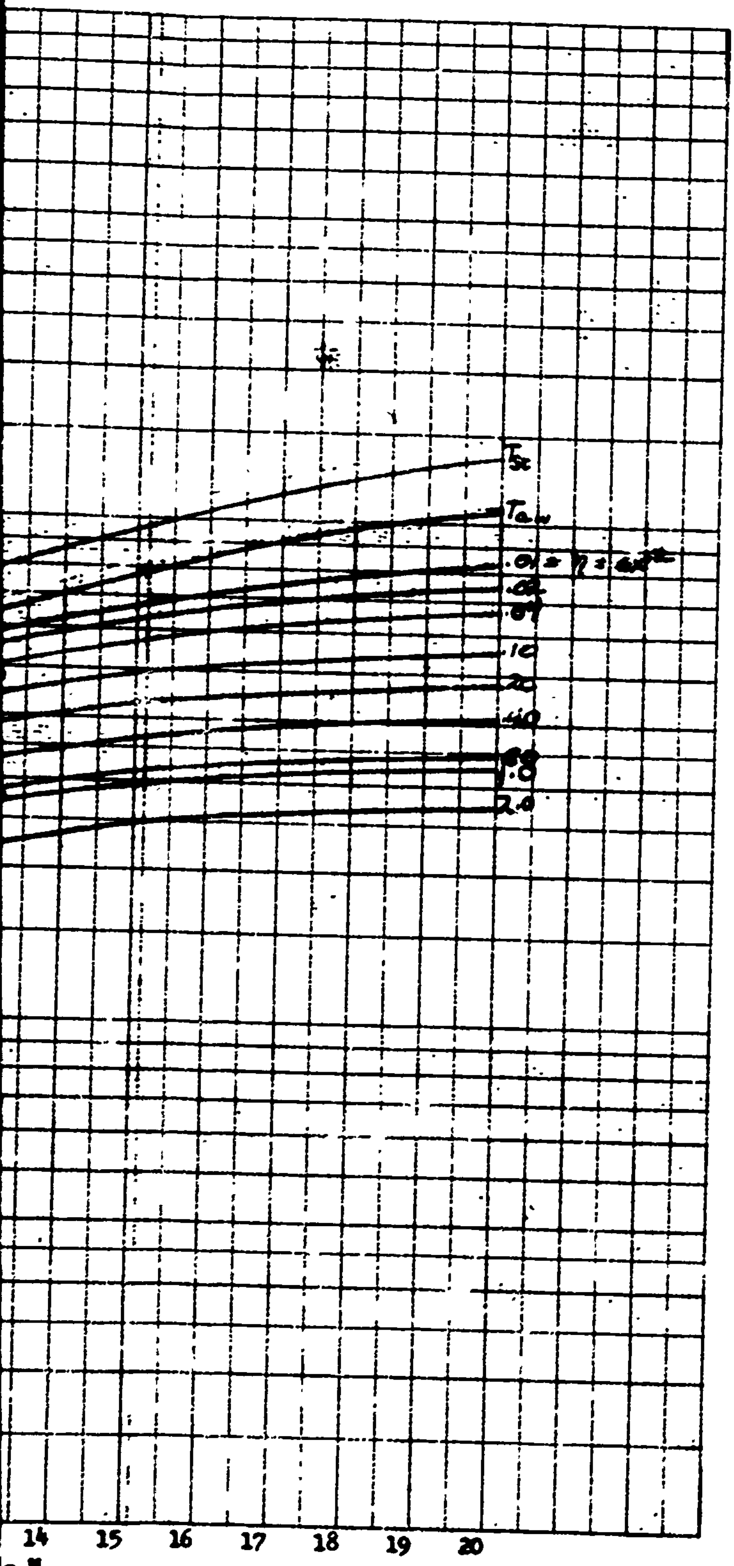
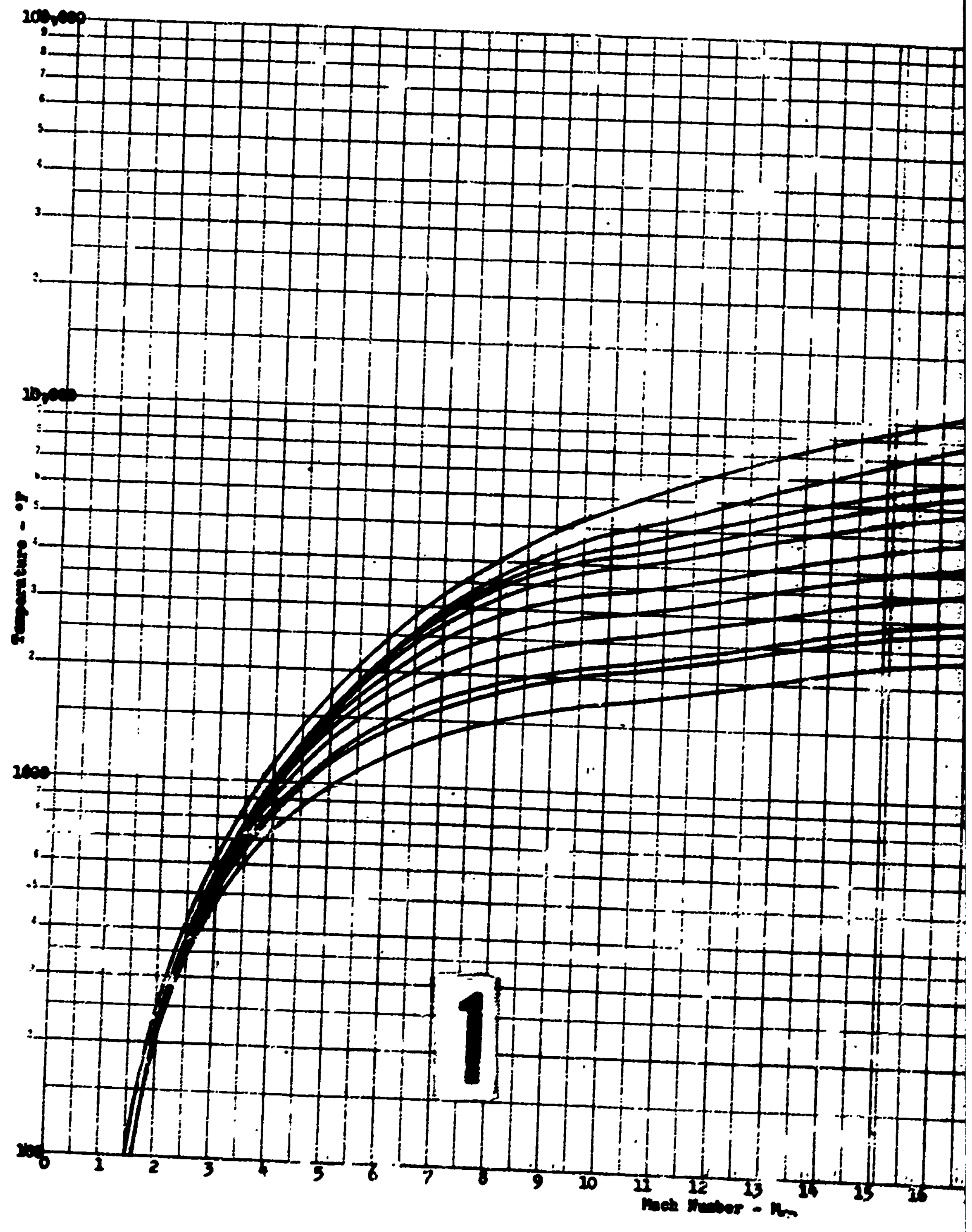


Figure 20 EQUILIBRIUM, STAGNATION, AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - TURBULENT BOUNDARY LAYER ALTITUDE = 50,000 FEET

3


100,000
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Temperature - °F
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16
Mach Number - M_∞
1

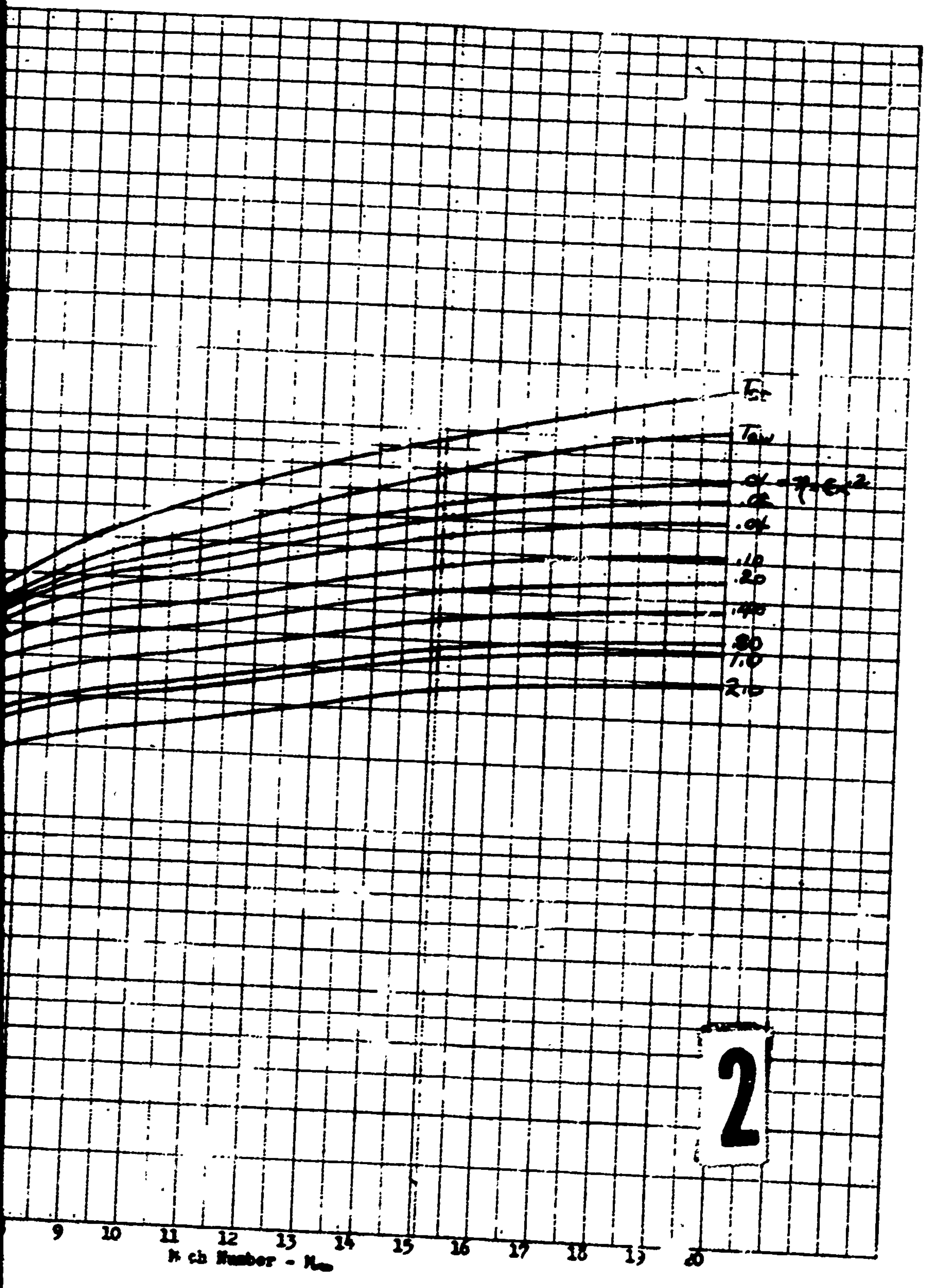


Figure 21 EQUIL..BRIUM, STAGNATI
WALL TEMPERATURE V
STANDARD DAY - TURBUL
ALTITUDE = 60,00

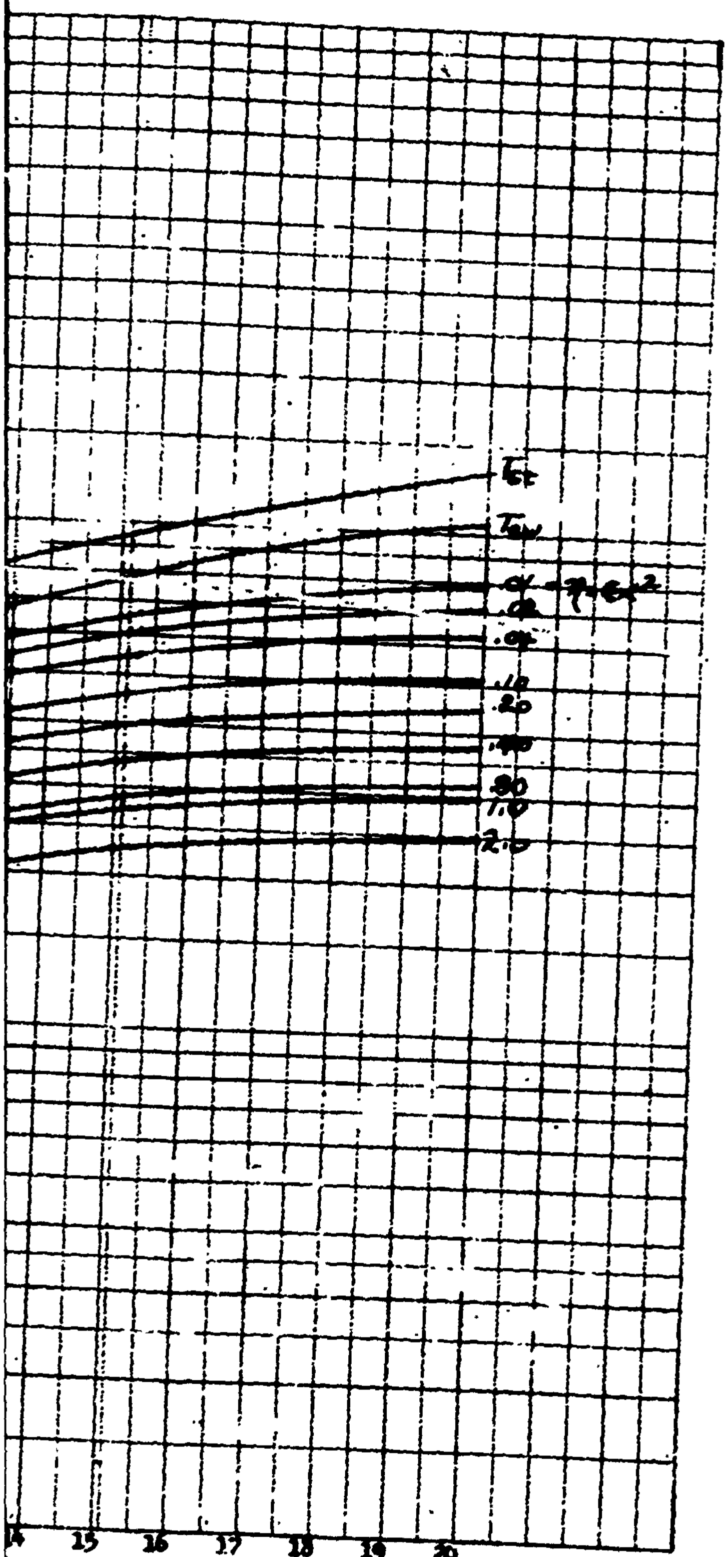
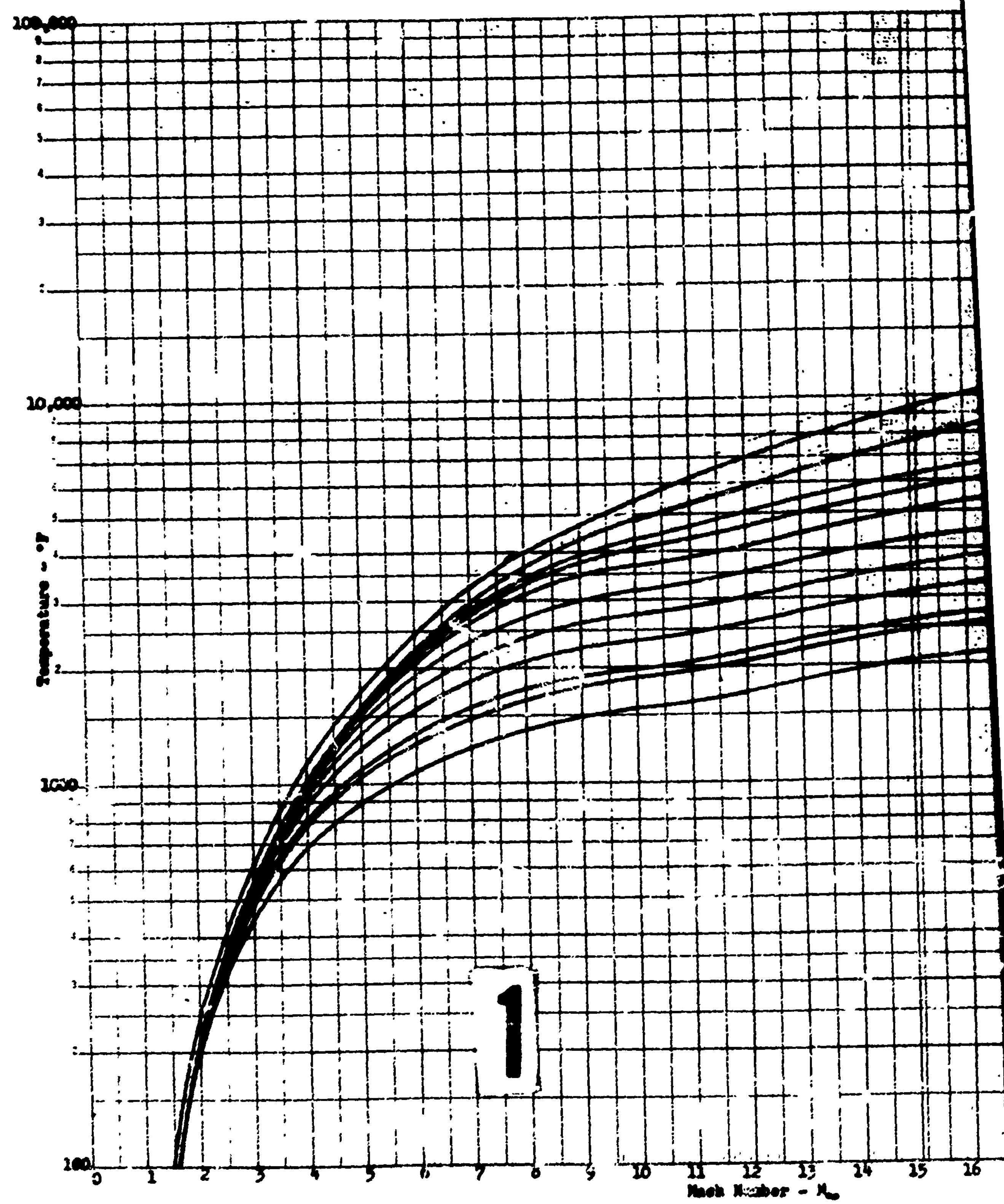


Figure 21 EQUILIBRIUM, STAGNATION, AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - TURBULENT BOUNDARY LAYER ALTITUDE = 80,000 FEET


100,000
10,000
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Temperature - °F
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16
Mach Number - M_∞

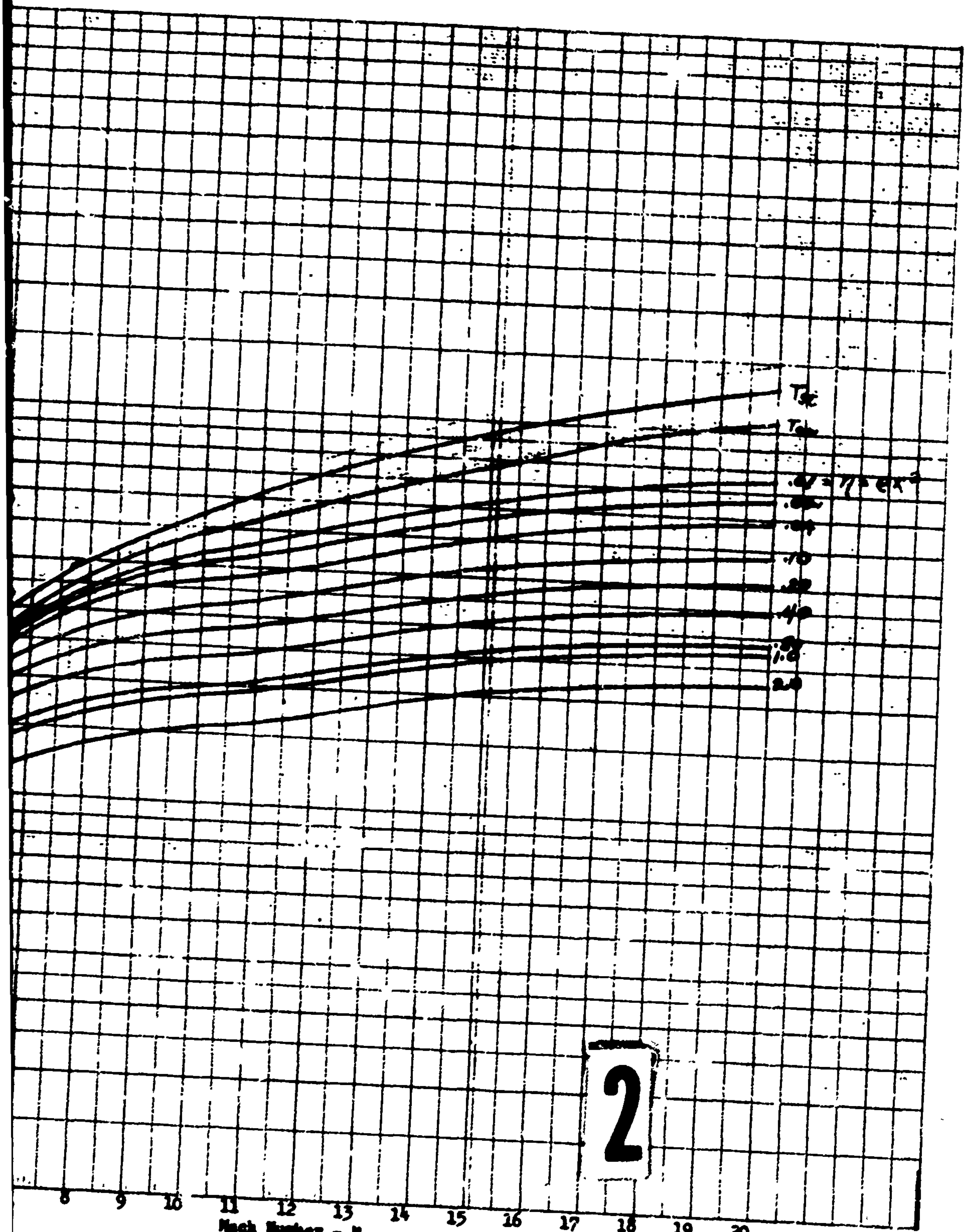


Figure 22 EQUILIBRIUM, STAG
WALL TEMPERATU
STANDARD DAY - T
ALTITUDE = 7

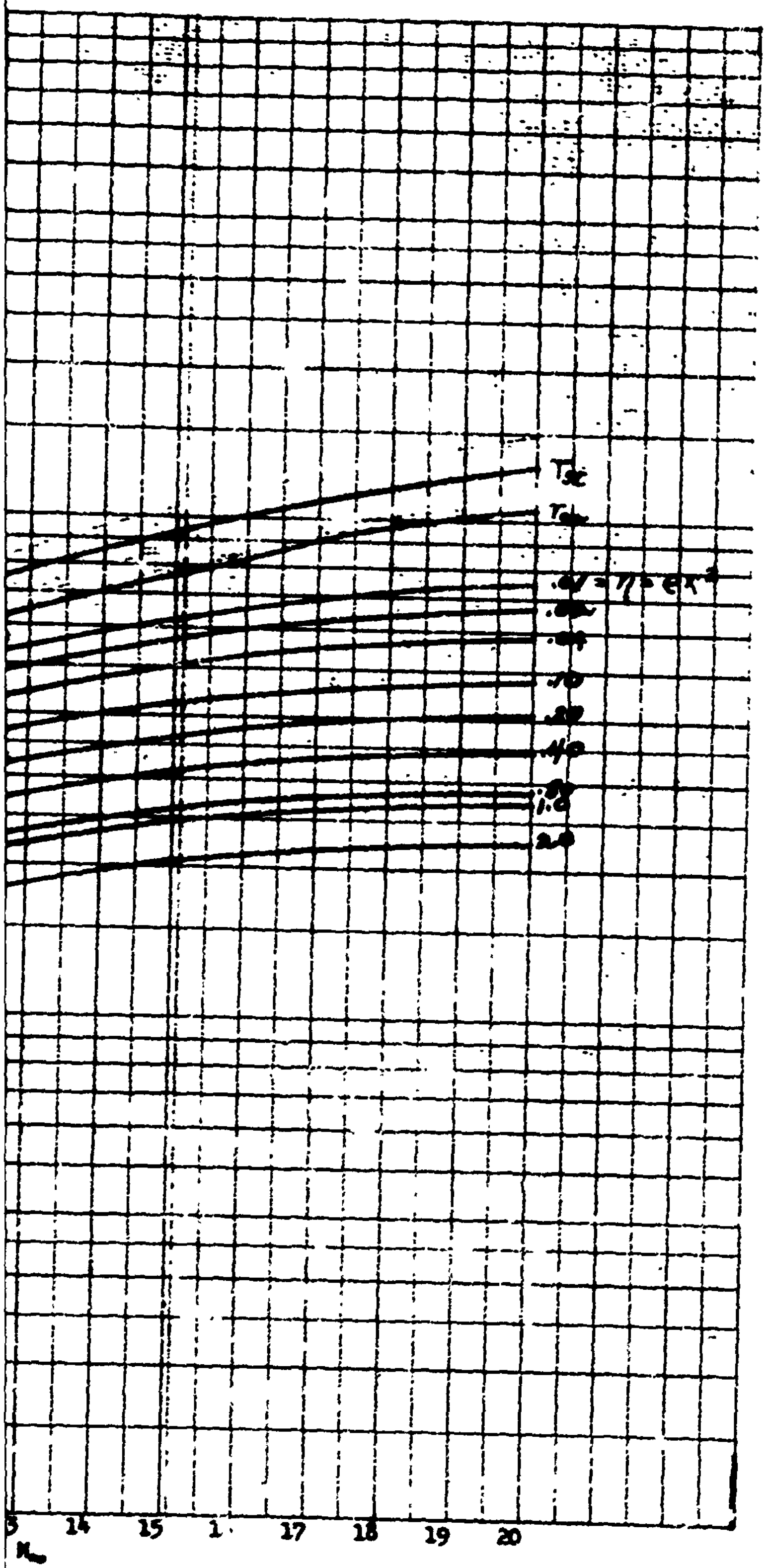
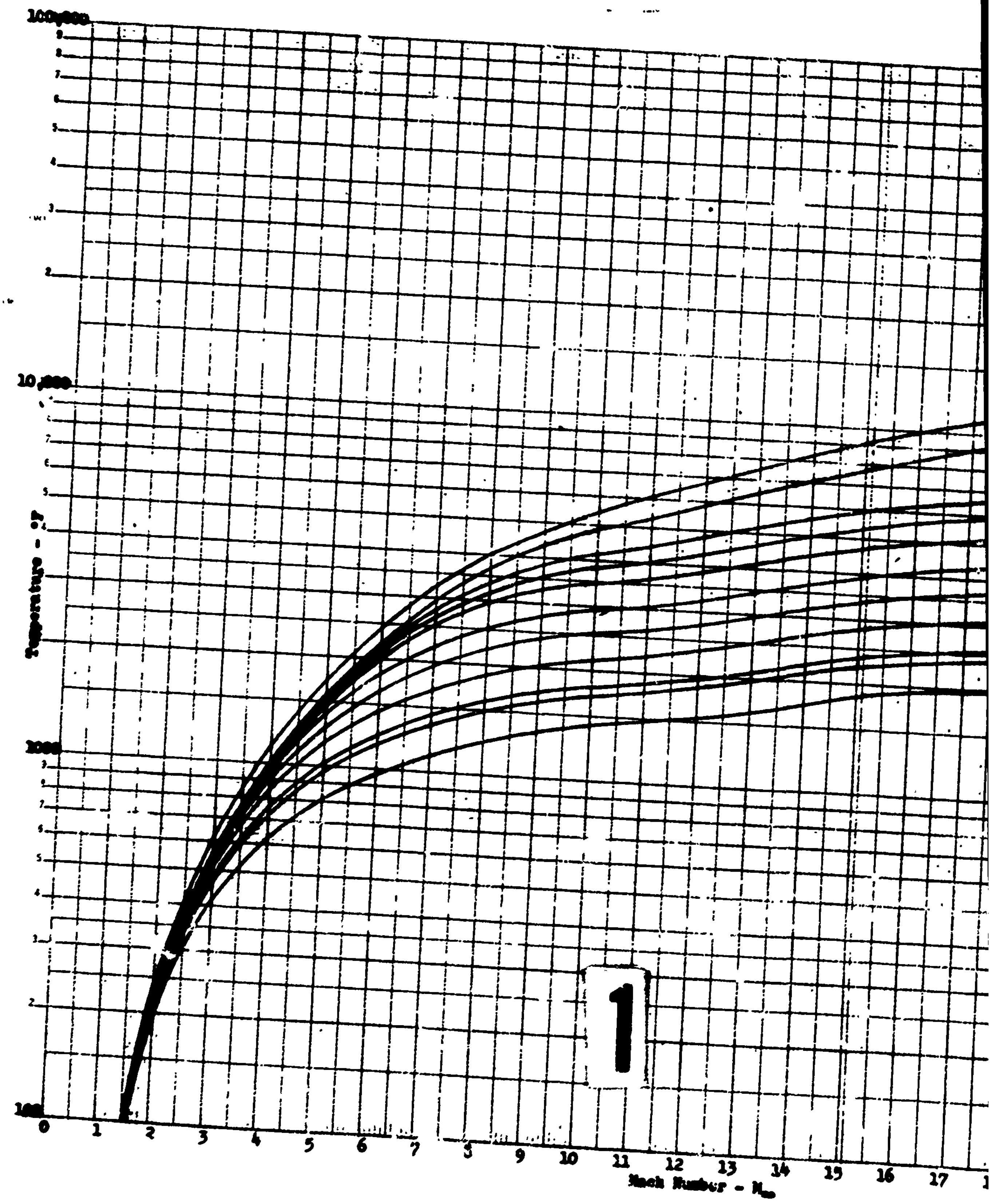


Figure 22 EQUILIBRIUM, STAGNATION, AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - TURBULENT BOUNDARY LAYER ALTITUDE = 70,000 FEET

3


100,000
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Temperature - °R
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17
Mach Number - M_∞
1

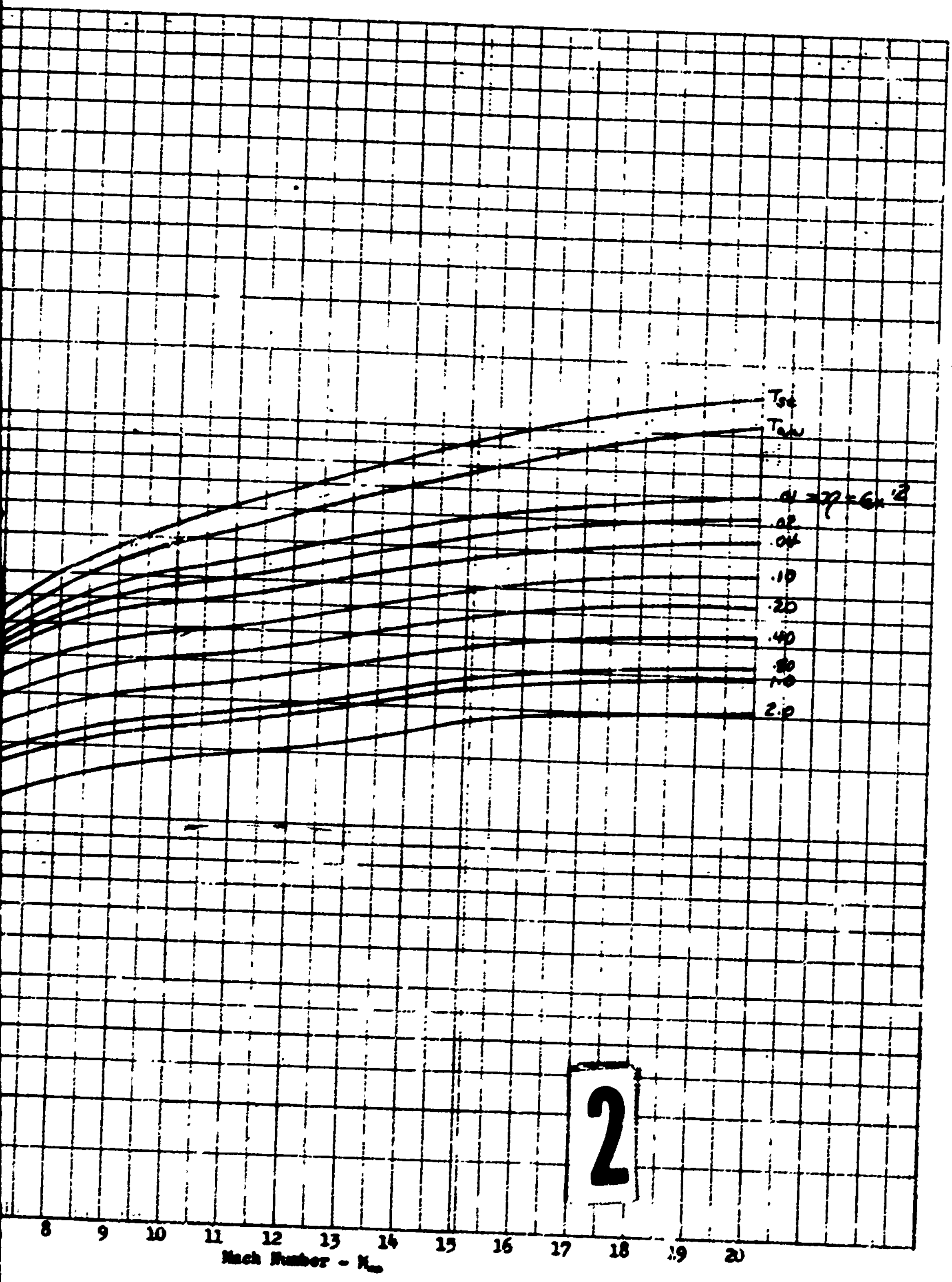


Figure 23 EQUILIBRIUM, STAGNAT
WALL TEMPERATURE
STANDARD DAY - TURB
ALTITUDE = 80,

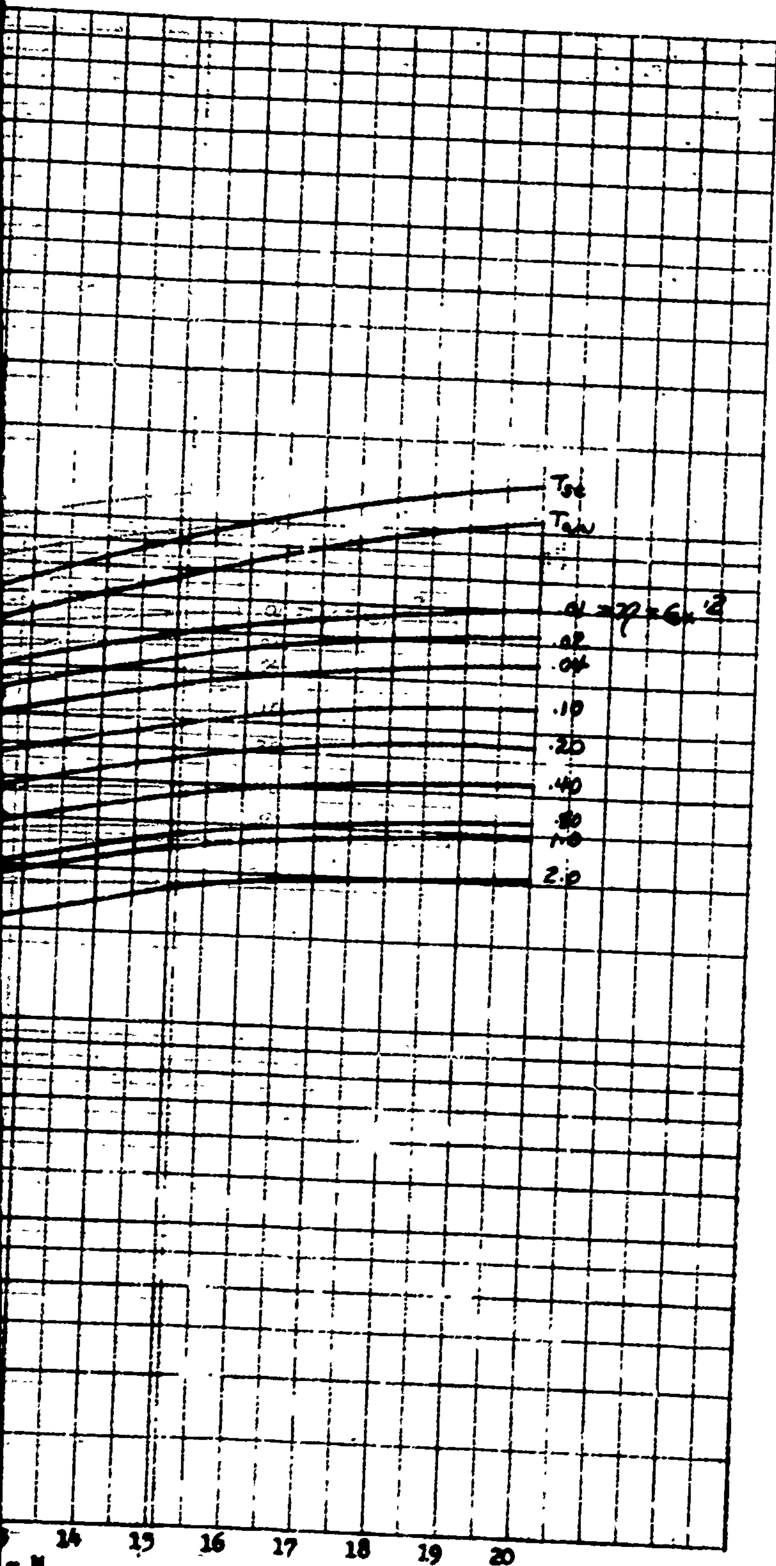
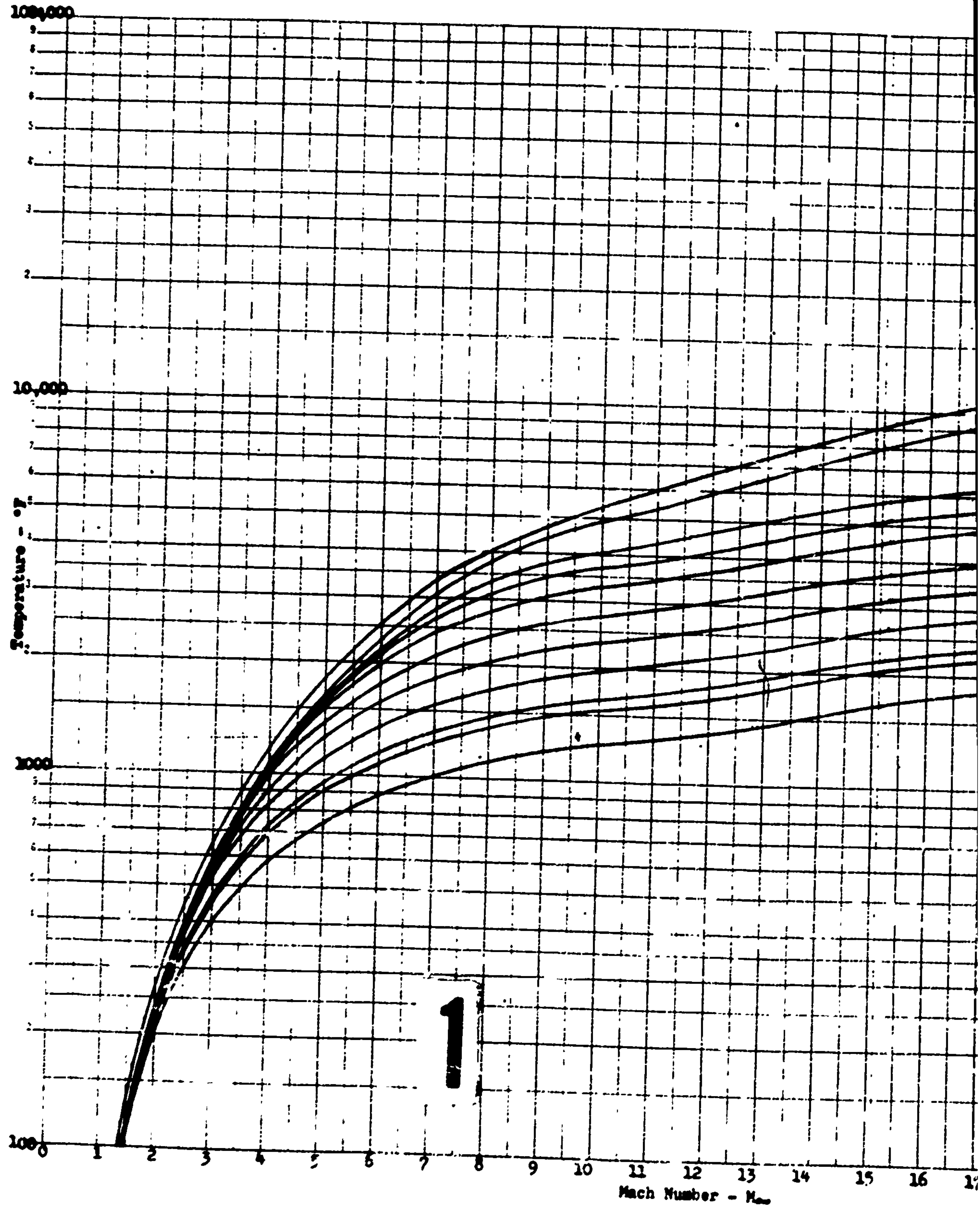


Figure 23 EQUILIBRIUM, STAGNATION, AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - TURBULENT BOUNDARY LAYER ALTITUDE = 80,000 FEET

3


100,000
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Temperature - °F
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16
Mach Number - M_∞

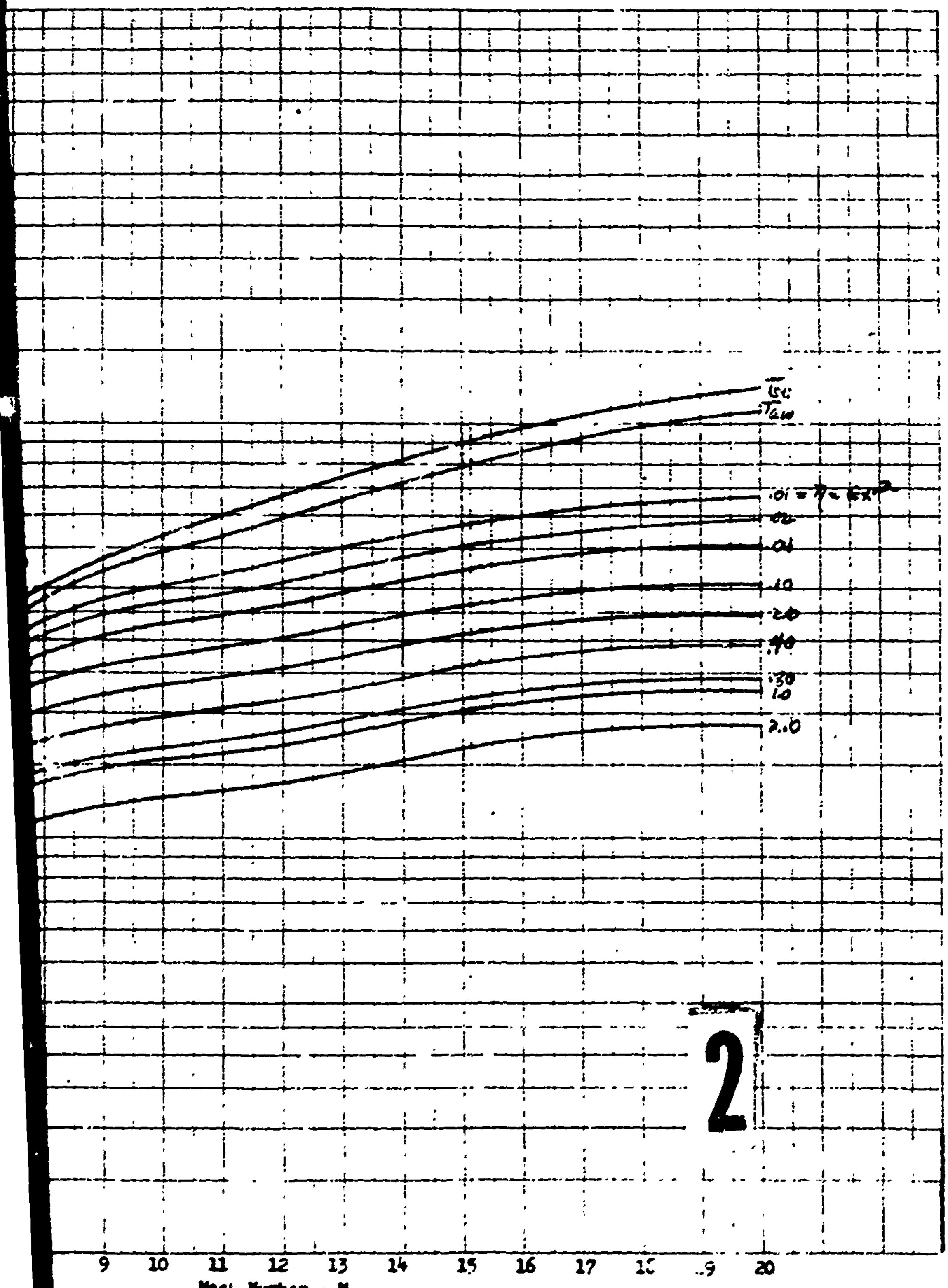


Figure 24 EQUILIBRIUM, STAGNATION, WALL T.MPERATURE VERS STANDARD DAY - TURBULENT ALTITUDE = 90,000

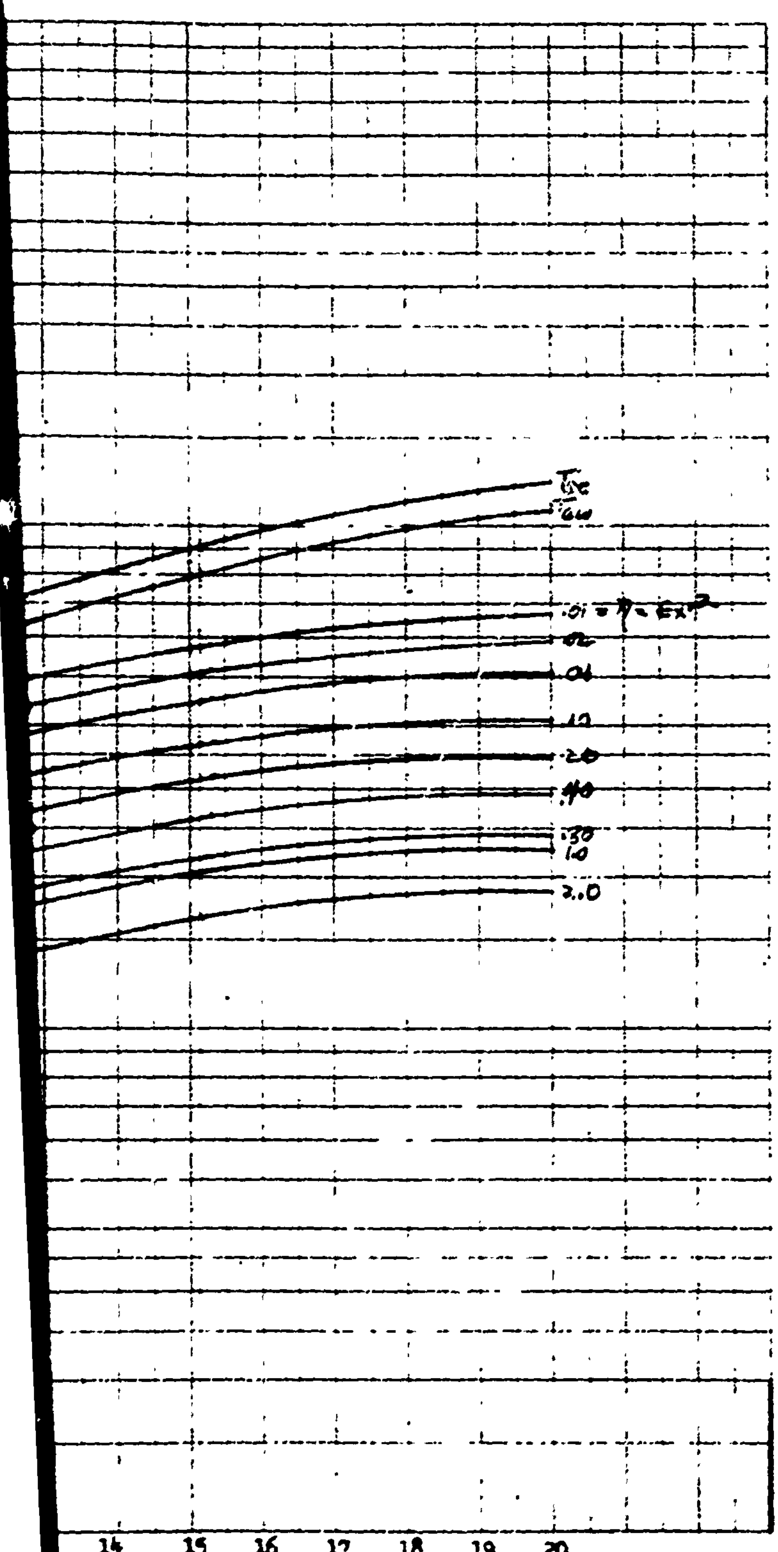
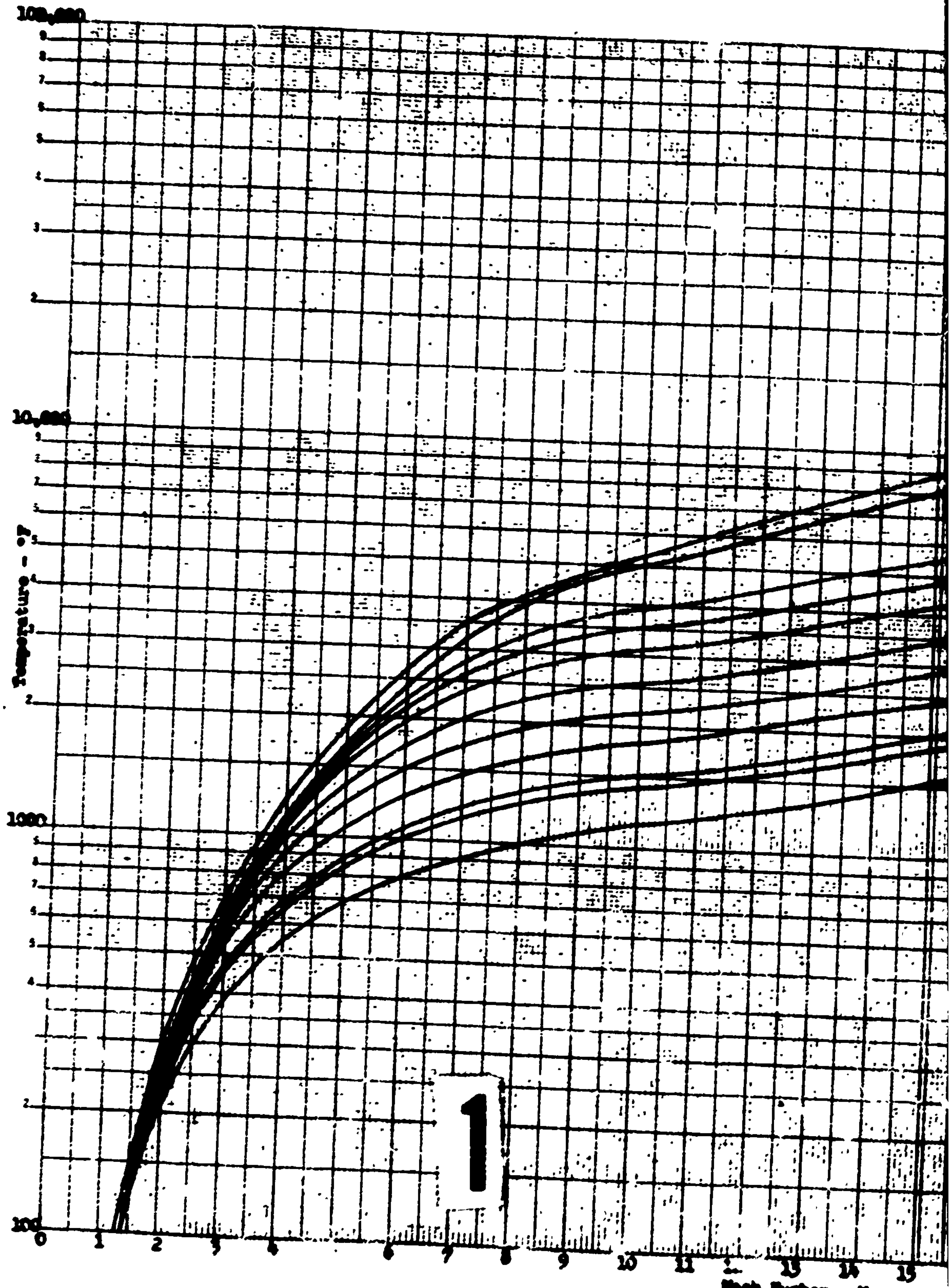


Figure 24 EQUILIBRIUM, STAGNATION, AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - TURBULENT BOUNDARY LAYER ALTITUDE = 90,000 FEET

3


100,000
10,000
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Temperature - °F
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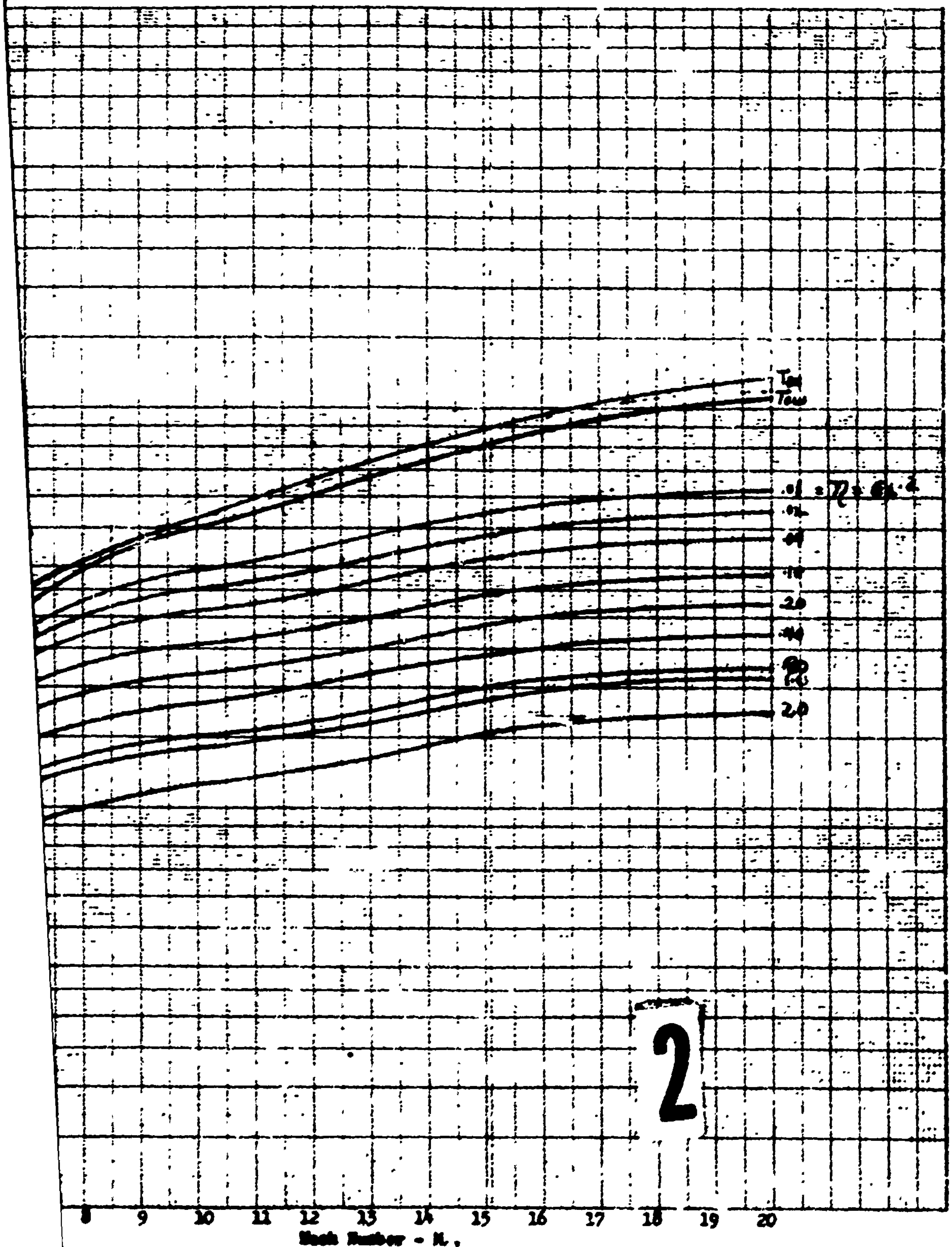
Mach Number - M_∞



2

Figure 25 EQUILIBRIUM, STAGN
WALL TEMPERATURE
STANDARD DAY - TUR
ALTITUDE = 100

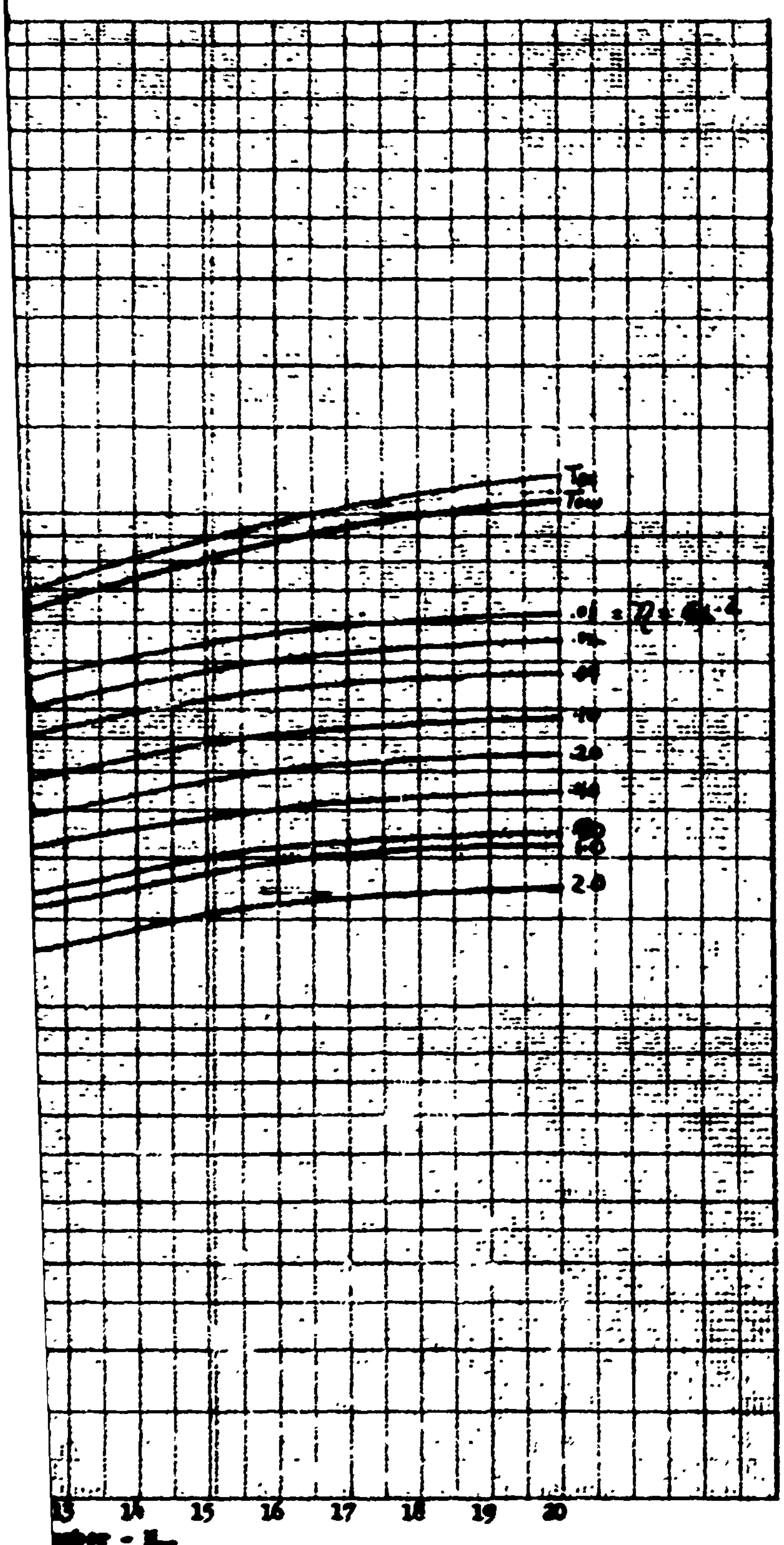


Figure 25 EQUILIBRIUM, STAGNATION, AND ADIABATIC WALL TEMPERATURE VERSUS MACH NUMBER STANDARD DAY - TURBULENT BOUNDARY LAYER ALTITUDE = [illegible]00,000 FEET

APPENDIX A

DEFINITION OF TERMINOLOGY

A - Surface area

a_L - Speed of sound based on local temperature

c_f - Skin friction coefficient

c_p - Specific heat of air at constant pressure

c_v - Specific heat of air at constant volume

g - Constant equal to 32.2

J - Constant equal to 778

h - Heat transfer coefficient

h' - Parameter equal to hx^{α}

H_{aw} - Adiabatic wall enthalpy

H_L - Local enthalpy

H' - Reference enthalpy

H_s - Stagnation enthalpy

k - Thermal conductivity

M_L - Local Mach number

M_∞ - Free stream Mach number

Nu - Nusselt number

Pr - Prandtl number

q_c - Convective heating rate

q_r - Radiative heating rate

R - Gas constant

R_x - Reynolds number based on distance from leading edge

S_t - Stanton number

T_{aw} - Adiabatic wall temperature

T_e - Equilibrium temperature

T_L - Local temperature

T_s - Effective temperature of space

T_{st} - Stagnation temperature

T_w - Wall temperature

V_L - Local velocity

x - Distance from leading edge, reference length

α - Constant, 0.5 for laminar flow, 0.2 for turbulent flow

γ - Ratio of specific heats, c_p/c_v

ϵ - Emissivity

η - Parameter equal to ϵx^{α}

ρ - Density

σ - Stefan-Boltzmann's constant = 0.4758×10^{-12} Btu/sec $ft^2\,{}^{\circ}R^4$

μ - Viscosity

APPENDIX B

DERIVATION OF FLAT PLATE HEAT TRANSFER COEFFICIENT

LAMINAR BOUNDARY LAYER

Blasius was the first to solve the differential equations for the incompressible laminar boundary layer on a flat plate. He did this by means of mathematical transformations, which resulted in a single ordinary non-linear differential equation which he solved numerically. His result was:

$$c_f = \frac{0.664}{R_x^{0.5}} \qquad \text{(a)}$$

From Ref. 4 the modified Reynolds analogy between skin friction and heat transfer is given by:

$$S_t = \frac{c_f}{2Pr^{0.666}} \qquad \text{(b)}$$

Combining Eqs. (a) and (b) results in:

$$S_t = \frac{0.332}{R_x^{0.5}\,Pr^{0.666}} \qquad \text{(c)}$$

The Nusselt number is defined as:

$$Nu = \frac{hx}{k} = S_t R_x Pr \qquad \text{(d)}$$

Substituting Eq. (c) into Eq. (d) results in:

$$\frac{Nu}{R_x Pr} = \frac{0.332}{R_x^{0.5} Pr^{0.666}}$$

$$Nu = 0.332\ R_x^{0.5}\ Pr^{0.333}$$

or

$$h = 0.332 \frac{k}{x} R_x^{0.5}\ Pr^{0.333} \qquad \text{(e)}$$

Equation (e) was reworked for computer solution in the following manner:

$$Pr = \frac{c_p \mu}{k}$$

Multiplying and dividing Eq. (e) by Pr gives:

$$h = 0.332 \frac{k}{x} R_x^{0.5}\ Pr^{-0.666} \frac{c_p \mu}{k} \qquad \text{(f)}$$

Using the definition of Reynolds number:

$$R_x = \frac{\rho V_L x}{\mu}$$

and expressing c_p as follows:

$$c_p - c_v = \frac{R}{J}$$

$$c_p\left(1 - \frac{1}{\gamma}\right) = \frac{R}{J}$$

$$c_p = \frac{\gamma}{(\gamma - 1)} \frac{R}{J}$$

Equation (f) can be expressed as:

$$h = \frac{0.332}{x^{0.5}} (\rho V_L)^{0.5} \left(\frac{\gamma}{(\gamma - 1)} \frac{R}{J}\right) Pr^{-0.666} \mu^{0.5} \quad (g)$$

Define:

$$h = \frac{h'}{x^{0.5}}$$

Then:

$$h' = 0.332 (\rho V_L)^{0.5} \left(\frac{\gamma}{(\gamma - 1)} \frac{R}{J}\right) Pr^{-0.666} \mu^{0.5} \quad (h)$$

Equation (h) was derived from incompressible flow relations. However, Rubensin and Johnson (Ref. 5) discovered that Eq. (h) would be valid for compressible flow if the property values, ρ, γ, Pr, and μ, were introduced at some reference temperature. T', which was somewhere between the temperature extremes encountered in the boundary layer. Because of the high temperatures considered in this report, it was necessary to use a reference enthalpy, H', rather than a reference temperature. The equation used to obtain H' was:

$$\frac{H'}{H_L} = 1 + 0.032 M_L^2 + 0.58 \left(\frac{H_w}{H_L} - 1\right) \quad (i)$$

Thus. Eqs. (h) and (i) were used to compute the h' needed in Eq. (4) in the text.

TURBULENT BOUNDARY LAYER

The Blasius empirical skin friction law (Ref. 6) for turbulent flow is:

$$\frac{c_f}{2} = \frac{0.0296}{R_x^{0.2}} \qquad (j)$$

However, Seban and Doughty (Ref. 7) concluded on the basis of subsonic flat plate experiments that Eq. (j) should be modified to:

$$\frac{c_f}{2} = \frac{0.0265}{R_x^{0.2}} \qquad (k)$$

Colburn (Ref. 8) concluded that the modified Reynolds analogy of Eq. (b) was applicable to turbulent as well as laminar flow. Therefore, combining Eqs. (b), (d), and (k) results in:

$$Nu = 0.0265\ (R_x)^{0.8}\ Pr^{1/3}$$

or

$$h = 0.0265\,\frac{k}{x}\,(R_x)^{0.8}\ Pr^{1/3} \qquad (l)$$

Reworking Eq. (l) for computer computation in the same fashion as the laminar case results in the following form:

$$h' = 0.0265\ (\rho V_L)^{0.8}\ \left(\frac{\gamma}{(\gamma - 1)}\right)\frac{R}{J}\,Pr^{-0.666}\ \mu^{0.2} \qquad (m)$$

Equation (m), like its laminar counterpart, was based on incompressible flow relations. However, it may be used for compressible flow if the fluid properties are evaluated at a reference temperature defined by Summer and Short (Ref. 9, Again, because of the high temperature region in which some of the values were obtained, the use of reference enthalpy was made. It is defined as:

$$H' = 0.45\ H_w + 0.55\ H_L(1 + 0.064\ M_L^2) \qquad (n)$$

Equations (m) and (n) supplied the h' needed by Eq. (4) in the text for the turbulent calculations.

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